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Pseudo Relative Permeability Functions. Limitations in the Use of the Frontal Advance Theory for 2-Dimensional Systems

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Abstract

To obtain a reliable characterization of complex reservoirs, they must be represented with fine scale models. However, detailed descriptions demand a huge amount of information and processing time. Therefore, less complex models are usually designed. To construct these "simple" models, pseudo relative permeability curves are developed. Pseudos reduce the number of dimensions of reservoir models, trying to reproduce the behavior of the fine scale system. Simplifying 2-dimensional (2D) systems to 1-dimensional (1D) models allows reservoir engineers to apply them to simple calculations and less complex reservoir simulations. Once reduced to 1D, models are usually analyzed by means of frontal advance theory.

In this paper, stratified systems under no-crossflow and under vertical equilibrium (VE) conditions were studied by means of numerical simulation (NS), analytical methods and simplified pseudo relative permeability functions (PRPF). When frontal advance theory was used, some restrictions were found to the application of pseudo functions. These limitations, if overlooked, can yield wrong conclusions.

Introduction

The use of analytical methods to predict natural reservoir performances is based on simplifications that enable reservoir engineers to apply simple models to describe complex geological structures.

The simplest inmiscible displacement model, initially developed by Buckley & Leverett[1] and later on reformulated by Welge[2], is the well-know frontal advance theory. This model was derived for lineal and continuous systems.

To apply Buckley & Leverett and Welge's equations to complex systems, it is necessary to reduce them to 1D models. This purpose is usually achieved by means PRPF [3,4]. These pseudo functions reproduce the global behavior of the original system in a 1D model. Then, once the appropriate pseudo functions have been generated, 1D models (x) can represent 2D geological structures (x-z). Both NS and analytical models frequently use the same strategy.

Prestigious authors as Dake[5,6] and Willhite[7] have taken for granted that having generated the PRPF, the frontal advance theory can be employed in the resultant 1D model. Thus an inmiscible displacement project can be easily studied.

In this paper it is shown that the use of PRPF to simplify 2D heterogeneous systems or 2D homogeneous systems deeply influenced by gravity forces, not always yield a 1D model that can be reliably analyzed with Buckley & Leverett and Welge's technique.

Theoretical Background: The Frontal Advance Theory

The frontal advance theory is a very valuable tool for reservoir engineers to study, in a simple manner, the performance of reservoirs under waterflooding.

Buckley & Leverett took the concept of fractional flow presented in 1941 by Leverett[8], that in the case of a waterflooding it is expressed as:

$$f_w = \frac{q_w}{q_w + q_o} \quad \text{...} (1)$$

Replacing the well-known Darcy's law for water and oil we obtain:

$$f_w = \frac{1 + \frac{kk_{ro}}{\mu_o}\frac{A}{q_t}\left(\frac{\partial P_c}{\partial L} - g\Delta\rho sin\alpha_d\right)}{1 + \frac{\mu_w}{\mu_o}\frac{k_o}{k_w}} \quad \text{............................} (2)$$

It is worth mentioning here that, for a given set of rock, fluids and flowing conditions, the fractional flow of water (f_w)

is solely a function of water saturation. Furthermore, as f_w is usually measured at the outlet face of the system, the corresponding S_w should be expressed at the same point.

In 1942 Buckley & Leverett presented the frontal advance equation:

$$\left(\frac{\partial L}{\partial t}\right)_{S_w} = \frac{q_t}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t \quad (3)$$

as a result of the application of the law of conservation of mass to the unidirectional flow of two inmiscible fluids (in the present case, oil and water) through a continuous and homogeneous porous media. This equation assumes the fluids and the porous media are incompressible. Eq. 3 states that a certain fixed water saturation moves through the porous media at a rate that is constant and proportional to the change in composition of the flowing stream caused by a small change in the saturation of the displacing fluid.

Some years later, in 1952, Welge went on to derive an equation that relates the average water saturation with the saturation located at the producing end of the system:

$$\overline{S}_w - S_{w2} = Q_i\left(1 - f_{w2}\right) \quad (4)$$

This very important equation states that knowing the cumulative injected pore volumes of water (Q_i), the water saturation and the fractional flow at the producing end (S_{w2} and f_{w2} respectively) it is possible to calculate the average water saturation and furthermore, the cumulative oil production.

Welge introduced another equation:

$$Q_i = \frac{1}{\left(\frac{\partial f_w}{\partial S_w}\right)_{S_{w2}}} \quad (5)$$

that relates Q_i with S_{w2}.

As mentioned before, Eq. 4 and 5 can be associated with the cumulative oil production during waterflooding. Before water breaks through at the end point, the volume of oil recovered is equal to the volume of water injected into the system:

$$N_p = \frac{W_i B_w}{B_o} \quad (6)$$

where:

$$W_i = Q_i V_p \quad (7)$$

Eq. 6 is not valid after breakthrough because a portion of the water injected is produced at the outlet end. This is equivalent to say that the system is producing with certain water cut. Then, under these conditions, the applicable equation to estimate the cumulative oil production is:

$$N_p = \frac{V_p\left(\overline{S_w} - S_{wc}\right)}{B_o} \quad (8)$$

Welge's equations are valid for 1D systems. However several authors as Dake and Willhite have pointed out that they can also be applied to complex systems. As an example the transformation of a 2D system into a 1D model can be quoted, for which these authors suggest using PRPF to preserve the effect of heterogeneity and non-uniform fluid distribution in the vertical direction.

In the case of a multi layer system, the equations involved in this process are[6]:

$$\overline{S}_{wn} = \frac{\sum_{i=1}^{n} h_i\phi_i\left(1 - S_{ori}\right) + \sum_{i=n+1}^{N} h_i\phi_i S_{wci}}{\sum_{i=1}^{N} h_i\phi_i} \quad (9)$$

$$\overline{k}_{rwn} = \frac{\sum_{i=1}^{n} h_i k_i k'_{rwi}}{\sum_{i=1}^{N} h_i k_i} \quad (10)$$

$$\overline{k}_{ron} = \frac{\sum_{i=n+1}^{N} h_i k_i k'_{roi}}{\sum_{i=1}^{N} h_i k_i} \quad (11)$$

These expressions calculate the pseudo relative permeability to water and oil and the water saturation at the outlet face of the system. In order to use them, the flooding order of the layers must be determined.

If there is a total of N layers, then there are N! ways in which they could be successively flooded. For the time being, however, we are concerned about the extreme conditions of pressure equilibrium among layers: complete communication or total absence of it. The former condition is called vertical equilibrium (VE)[3,9]. Under this condition it is assumed that water segregates instantaneously as it enters the porous media, owing to the difference between water and oil densities. Consequently the flooding sequence is from the bottom layer to the top. If contrarily the layers are isolated from one another, in a way that there is no cross-flow among them (no cross-flow) the flooding order is determined by the velocity of water advance in each, that is:

$$v_i \alpha \frac{k_i k'_{rwi}}{\phi_i\left(1 - S_{ori} - S_{wci}\right)} \quad (12)$$

Hence, the N layers will flood in decreasing order of their calculated velocities.

Description of the calculations developed

This study investigates the waterflooding performance of different kind of reservoirs and flowing conditions. To do so, the frontal advance theory (Welge) was compared with an analytical method based on Darcy's law (Darcy). In addition to this, NS was used to validate the obtained results.

The study was performed on the following reservoir models:

Example 1 - Homogeneous in VE and mobility ratio (M) equal to 0.3.

Example 2 - Homogeneous in VE and M=3.0.

Example 3 - Stratified in VE and M=1.

Example 4 - Stratified in no cross-flow and M=1.

Table 1 lists the rock and fluid properties for examples 1 and 2. Table 2 and Fig. 1 show the properties of the models used for example 3 and 4.

Welge method was applied using the equations mentioned in the previous section.

The PRPF used are plotted on Fig. 2 and 3. In the case of examples 1 and 2 (Fig. 2), the curves are straight lines that connect the end points of the original rock relative permeability curves. In examples 3 and 4 (Fig. 3), the PRPF are identical due to the fact that the layers were systematically ordered with the most permeable layer on bottom. Therefore, the flooding order would be the same for both of them.

In these two later examples, pseudos do not recognize the difference between VE and no cross-flow conditions. The reason for this is that their calculations are based on punctual water saturation at the outlet face of the system and this property varies in the same manner for examples 3 and 4 during waterflooding. However the difference between the two flowing conditions is revealed in Fig. 4. Under VE, while a layer is being flooded there is no water entrance to any other above it. On the other hand, under no cross-flow, while a layer is being flooded the others admit a volume of water proportional to the velocity calculated with Eq. 12. In the case of example 4, the relative entrance of water is only proportional to absolute permeabilities, since any of the other properties involved in Eq. 12 are the same for all layers.

Examples 3 and 4 were developed for M=1. Such an assumption was made to simplify PRPF calculation.

Next, examples 1 to 4 were analyzed by means of Darcy's law, in a multiple-step sequence. The studies were performed taking into account the corresponding flowing conditions, for each one of the 2D selected models. It was supposed that the fluid saturation of the flooded zone changed from connate water to residual oil, with no in-between values. Besides, it was assumed that fluids and porous media were incompressible and there was no mass transfer between phases.

Finally, for each developed example the obtained results were compared (Welge vs. Darcy), with the aid of f_w vs. S_w and $N_p(V_p)$ vs. Q_i plots. According to the methodology used here, Darcy method was accepted as the most suitable to represent what really happens in the 2D models, under the assumed flowing condition.

Discussion of Results

Figs. 5 through 8 depict f_w vs. S_w plots. According to model definition, these curves are only function of mobility ratio, being the same for Welge and Darcy calculations. Note that for example 1, the plot obtained is characteristic of a 1D piston like displacement. Clearly this result is in direct contradiction to the VE condition initially assumed for this model.

Figs. 9 through 11 show $N_p(V_p)$ vs. Q_i obtained from Welge and Darcy methods, for examples 1 through 3. At first sight, it can be pointed out that both methods yield different results. When Darcy is applied, the necessary Q_i to produce all the mobile oil are greater than the ones obtained using Welge. Thus, it can be concluded that the latter assumes that the displacement is more efficient than it really is. An error of this magnitude would be very dangerous if we were using this method to predict the waterflooding performance of a real reservoir.

Then, bearing in mind what was pointed out in the previous paragraph, it can be concluded that identical f_w curves do not guarantee identical reservoir performance forecasts, when using different calculation methodology. Since Darcy method must be considered as the valid one, it can be concluded that the availability of f_w curve does not guarantee the applicability of Welge formulism to predict reservoir performances.

$N_p(V_p)$ vs. Q_i for example 4 is illustrated in Fig. 12. Both methods provide identical results. Hence, it can be concluded that in this case Welge method is suitable to study a 2D system reduced to a 1D model with the assistant of PRPF.

Finally, Fig. 13 depicts the comparison of $N_p(V_p)$ vs. Q_i curves obtained from Darcy method in examples 3 and 4. The different results are coherent with the flowing conditions assumed. Example 3 (under VE), shows an earlier breakthrough than example 4 (under no cross-flow), as it was expected. This can be easily visualized in Fig. 4, where it can be seen too that equal water saturations at the outlet face correlates with different average water saturations and so different cumulative oil productions. However, both examples provide the same PRPF and then the same results when applied Welge method.

As outlined before this is due to the fact that PRPF are calculated at the producing end, where the fluid distribution of 3 and 4 models are identical at any layer breakthrough.

Previous results were supported by NS, run at different flow regimes on the reservoir models presented. At low flow regimes, results were comparable to Darcy calculations and notably different from Welge’s. These additional results sustain the conclusion that Welge's formulism is not adequate to describe simplified 2D models where gravity force is significant. This conclusion could be extrapolated to 3-dimensional (3D) systems.

Conclusions

The results of this study lead to the following conclusions:

1. When gravity force is acting on a reservoir under waterflooding and PRPF are used to reduce 2D systems to 1D models, it is not valid to apply Welge technique to the resultant fractional flow curves.

2. In these mentioned cases, production forecasts can only be obtained through analytical models or NS that respects the 2D or 3D drive mechanism.

Nomenclature

A = area normal to the direction of movement
B = fluid volume factor
f = fractional flow of water and oil
g = acceleration due to gravity
h = layer thickness
k = absolute permeability
k_r = relative permeability
$\bar{k}_r$ = pseudo relative permeability
k'_r = end point relative permeability
L = distance along direction of movement
N = total number of layers
n = number of layers flooded
N_p = cumulative oil production
P_c = capillary pressure = $P_o - P_w$
q = fluid flow rate
Q_i = pore volumes of cumulative injected fluid
q_t = total fluid rate
S = saturation
$\bar{S}$ = average saturation
S_{ro} = residual oil saturation
S_{wc} = connate water saturation
t = time
V_p = pore volume of the system
W_i = volume of water injected
α_d = angle of the formation dip to the horizontal
$\Delta\rho$ = water oil density difference = $\rho_w - \rho_o$
ϕ = porosity
μ = viscosity

Subscripts

n = number of layers flooded
o = oil
w = water
2 = refers to outlet end location

Acknowledgments

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References

1. Buckley, S.E. and Leverett, M.C.: "Mechanism of Fluid Displacement in Sands," Trans., AIME (1942) 146, 107-16.
2. Welge, H.J.: "A Simplified Method for Computing Oil Recovery by Gas or Water Drive," Trans., AIME (1952) 195, 179-86.
3. Coats, K.H., Dempsey, J.R. and Henderson, J.H.: "The Use of Vertical Equilibrium in Two-Dimensional Simulation of Three-Dimensional Reservoir Performance," Soc. Pet. Eng. J. (March 1971) 63-71.
4. Hearn, C.L.: "Simulation of Stratified Waterflooding by Pseudo Relative Permeability Curves," JPT (Jul. 1971) 805-13.
5. Dake, L.P.: *Fundamentals of Reservoir Engineering*, Elsevier Scientific Publishing Co., New York City (1978) 365.
6. Dake, L.P.: *The Practice of Reservoir Engineering*, Elsevier Scientific Publishing Co., New York City (1994) 370-1.
7. Willhite, P.G.: *Waterflooding*, SPE Textbook Series, Richardson Texas (1986) 151-3.
8. Leverett, M.C.: "Capillary Behaviour in Porous Solids," Trans., AIME (1941) 142, 152-69.
9. Coats, K.H., Nielson, R.L., Terhune, M.H. and Weber, A. G.: "Simulation of Three-Dimensional, Two Phase Flow in Oil and Gas Reservoirs," Soc. Pet. Eng. J. (Dec. 1967) 377-88.

SI Metric Conversion Factors

cp × 1.0* E–03 = Pa · s
ft^3 × 2.831 685 E–02 = m^3
md × 9.869 233 E–04 = μm^2
psi × 6.894 757 E+00 = kPa

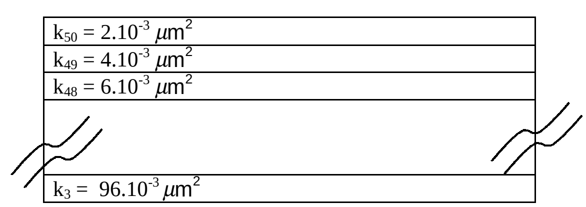
*Conversion factor is exact.

TABLE 1--RESERVOIR ROCK AND FLUID PROPERTIES-HOMOGENEOUS MODELS

Water viscosity, 10^{-3} Pa · s	1.0
Oil viscosity, 10^{-3} Pa · s	
(M=0,3)	1.0
(M=3)	10.0
Initial Water Saturation, fraction	0.35
Residual Oil Saturation, fraction	0.20
k, μm^2	0.11
k'_{ro} (S_{wc}), fraction	1.0
k'_{rw} (S_{ro}), fraction	0.3
Porosity, fraction	0.25
Capillary pressure, kPa	0
Angle of the formation dip to the horizontal, degree	0
Mobile oil volume, 10^{-6} m^3	112.5

TABLE 2--RESERVOIR ROCK AND FLUID PROPERTIES-HETEROGENEOUS MODELS

Number of layers	50
Water viscosity, 10^{-3} Pa · s	1.0
Oil viscosity, 10^{-3} Pa · s	3.3
Initial Water Saturation, fraction	0.35
Residual Oil Saturation, fraction	0.20
k'_{ro} (S_{wc}), fraction	1.0
k'_{rw} (S_{ro}), fraction	0.3
Porosity, fraction	0.25
Capillary pressure, kPa	0
Angle of the formation dip to the horizontal, degree	0
Mobile oil volume, 10^{-6} m^3	562.5



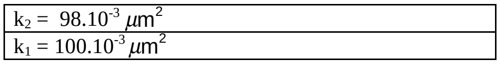


Fig. 1--Heterogeneous Models - All arranged layers with identical h, S_{wc} and S_{ro}.

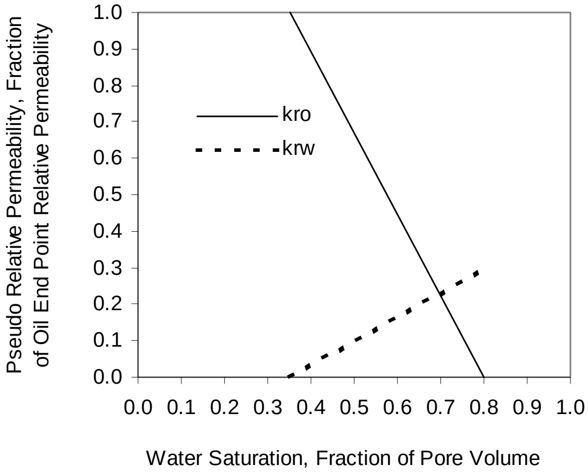


Fig. 2--Pseudo Relative Permeability Curves - Homogeneous Models.

Fig. 5--Welge vs. Darcy. Pseudo fractional flow comparison for example 1.

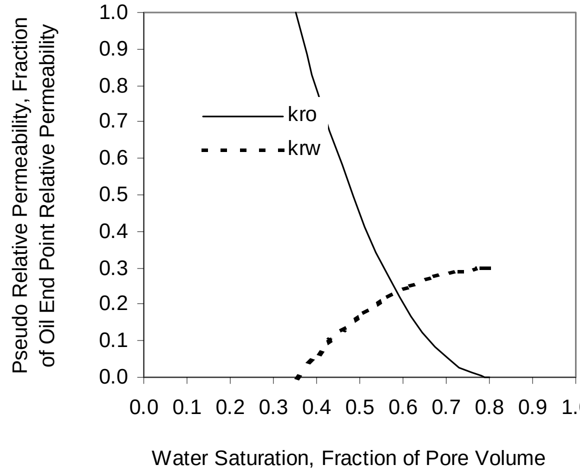


Fig. 3--Pseudo Relative Permeability Curves - Heterogeneous Models.

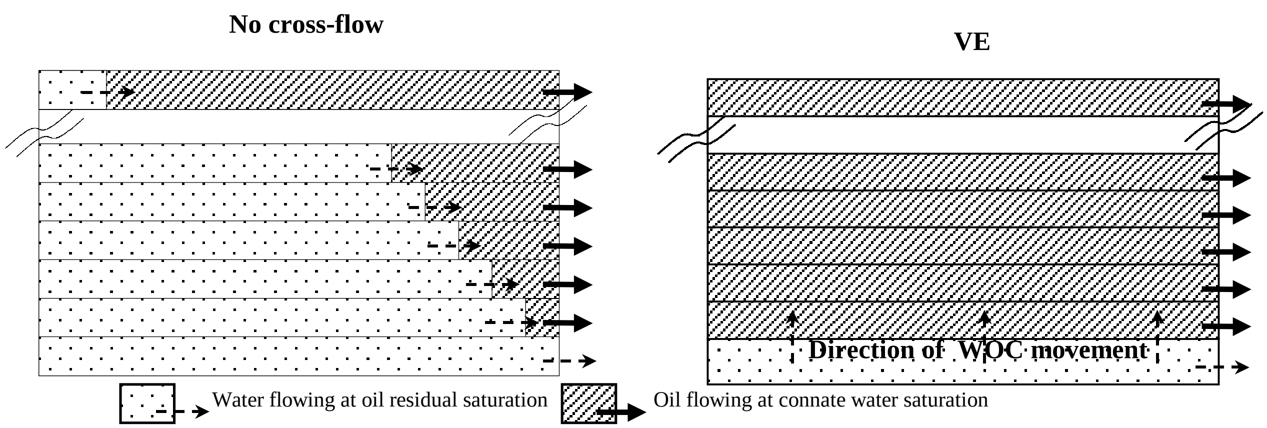
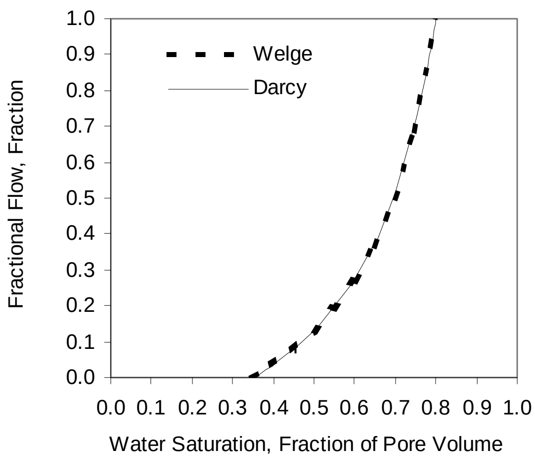


Fig. 4--Flowing conditions analyzed.



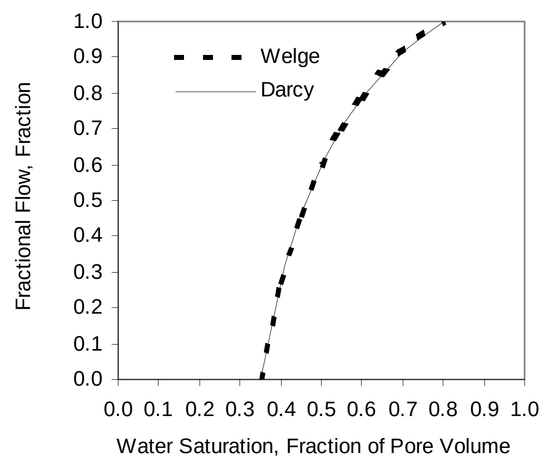


Fig. 6--Welge vs. Darcy. Pseudo fractional flow comparison for example 2.

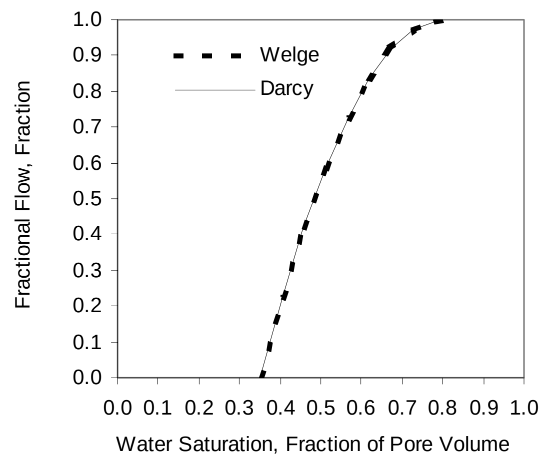


Fig. 11--Welge vs. Darcy. Np(Vp) vs. Q_i comparison for example 3.

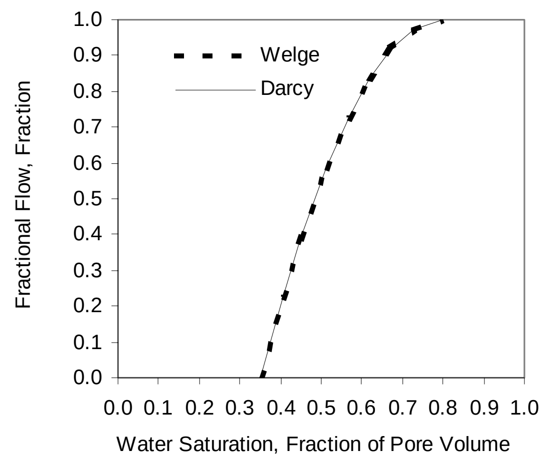
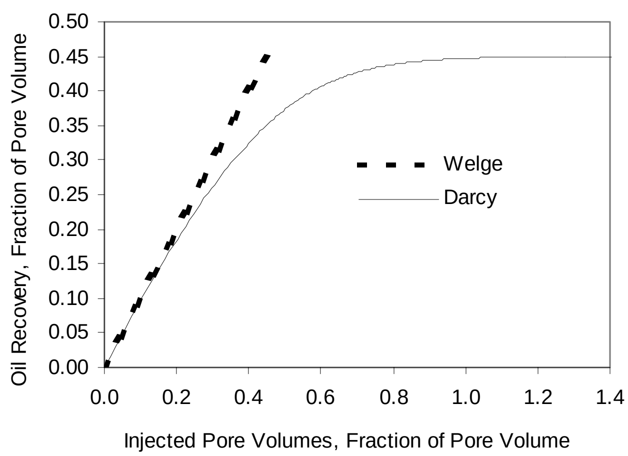


Fig. 7--Welge vs. Darcy. Pseudo fractional flow comparison for example 3.

Fig. 8--Welge vs. Darcy. Pseudo fractional flow comparison for example 4.



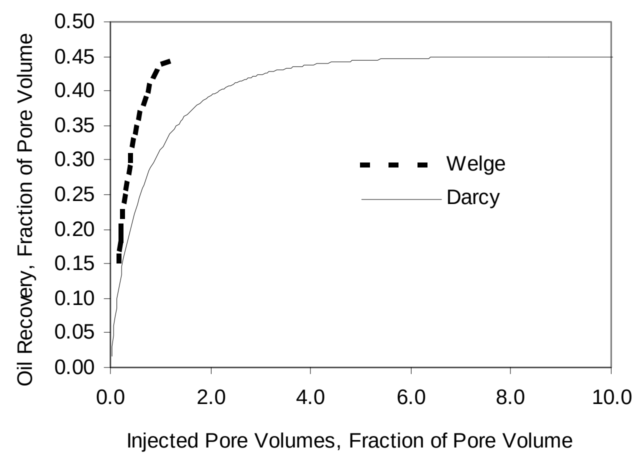
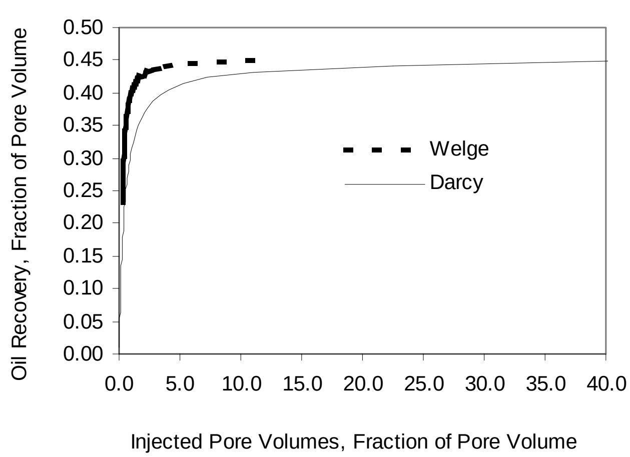


Fig. 9--Welge vs. Darcy. Np(Vp) vs. Q_i comparison for example 1.

Fig. 10--Welge vs. Darcy. Np(Vp) vs. Q_i comparison for example 2.



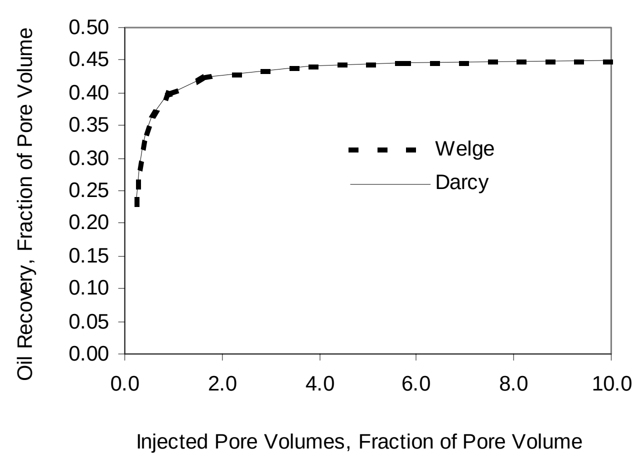


Fig. 12--Welge vs. Darcy. Np(Vp) vs. Q_i comparison for example 4.

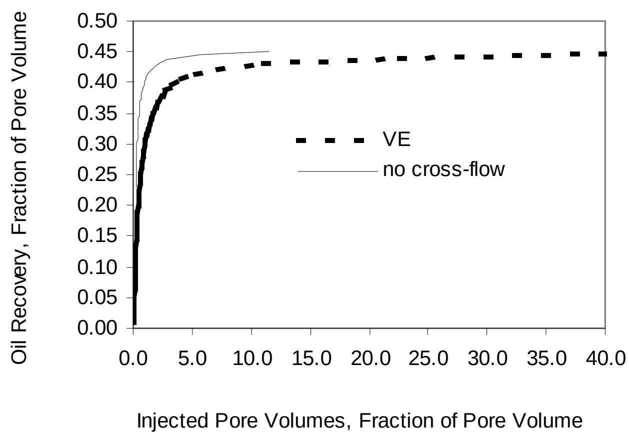


Fig. 13--Darcy method. Cases comparison: Np(Vp) vs. Q_i for example 3 (VE) and 4 (no cros-flow).