Chapter IV

"…
In one way or another, and indeed now for almost a Century, the voluminous literature on the subject has substantially been influenced by the evolving understandings about the meanings and applicability of what today still remains to be a perplexing (rather than an ubiquitous) relative permeability concept
…"

W. Rose - SPE 57442 - 1999.

INCONSISTENCIES

Over the past 60 years a vast number of papers have been published, analyzing or trying to solve different aspects of measurement, calculation or scaling up of relative permeabilities. This fact alone indicates we are facing a controversial subject with a complex solution. Furthermore, as controversy currently continues, we can conclude that the overall problem has not yet been solved.

The introductory quote, related to this controversy and coming from one of the most renowned experts on the subject is rather suggestive. It was taken from a 1999 publication[1] where W. Rose analyzes how his own opinion on the concept of relative permeability evolved through his publications, from 1948 on. In addition to the opening quote, another paragraph is especially significant in that paper:

> *"… Even so, the simple aim held by the Author of the present paper is to show that further clarifications still are urgently needed. And the focus of major importance now should have to do with how best to measure and to apply relative permeability data whenever simulation studies of reservoir transport processes are to be undertaken."*

From a historical and conceptual point of view, the discussion held over the years has been critical to the following main subjects:

- ✓ Relative permeability measurement methodology.
- ✓ Calculation methodology.
- ✓ Influence of heterogeneities.
- ✓ End point determination.
- ✓ Viscous fingering and instability of displacement front.
- ✓ Correlations.
- ✓ Influence of wettability.
- ✓ Three-phase displacements.
- ✓ Definition, validation and use of relative permeability pseudo-functions.

Facing this scenario, it is natural for a reservoir engineer –who routinely uses relative permeability curves- to feel discouraged and confused when carrying out modeling tasks involving fluid movement in the reservoir. The absence of a simple and reliable methodology to obtain these curves is uncomfortable, even paradoxical, since they are the variables having the greatest influence on fluids production.

In fact, as shown in this book, the problem associated with the relative permeability concept is much deeper than just better measurement or scaling-up procedures. At the same time, fortunately, the solution is simpler than what the debate seems to indicate. At least, it does not involve increasingly complex measurement equipment or calculation tools. As we will see, the solution involves better understanding of the operative forces in the reservoir and the adequate description of heterogeneities of the geological trap. And these two tasks are within the typical and traditional job assigned to reservoir engineers.

But..., in order to come up with the right solution it is necessary to start by eradicating some deeply rooted conceptual errors. Trying to eliminate them is not easy since these errors were born together with the relative permeability concept.

A conceptual analysis, demonstrating the inadequacy of relative permeability curves to describe fluid production in oil and gas reservoirs, was just discussed on Chapter lll.

The developments made in Chapter III can be summarized in the following logical chain.

1. Relative permeabilities quantify fluids **conduction** capacity in multiphase flows.
2. There are only two situations in which the multiphase conduction capacity has a defined value, namely:
   2.1. **Steady state flow**. This case recreates monophasic flow conditions, where: **conduction** = **injection** = **production** for each fluid.
   2.2. **At any single point on unsteady-state flow**. **Conduction** capacity varies from point to point, and equivalence between **injection**, **conduction** and **production is** broken in any finite volume
3. All real displacements of interest for the oil and gas industry, involve fluids **production** in unsteady state systems.

As a consequence, relative permeabilities cannot adequately model fluid **production** in any real system of interest[2].

In spite of how significant the previously analyzed restriction may be, this chapter will show it is not the only one related to the relative permeability concept.

The basic thesis on this chapter may be summarized as follows:

When fluid distribution is influenced by forces other than those associated with viscous displacement, relative permeability curves lose their applicability to describe fluid movement in the reservoir.

In other words, the analysis of this chapter focuses on the fact that, when capillary or gravity forces significantly influence fluid distribution in the reservoir, the relative permeability concept, and the equations derived therefrom - using the frontal advance theory- lose their physical meaning.

Two examples have been developed to demonstrate the above thesis.

The first example demonstrates a severe limitation on the use of pseudo functions together with fractional flow curves. It shows that, contrary to very common practices, when gravity forces are present, the frontal advance theory cannot be extended to the description of two-dimensional systems, even if made simpler by converting them into one-dimension models by using relative permeability pseudo functions.

The second example uses a very simple model to show that two porous media may have the same relative permeability set of curves but two totally different production histories. However astonishing this example may seem, it is based on the use of routine calculation techniques. In this example, to characterize a heterogeneous medium, the so-called relative permeability pseudo-functions are obtained, and they are taken as the relative permeability equivalents for the entire system.

FIRST EXAMPLE.
The fractional flow curve and the relative permeability pseudo functions

The frontal advance theory[3,4] describes immiscible, viscous-dominated displacements in uniform, one dimensional porous media geometry. Two significant conceptual results are obtained when developing this theory:

- ✓ Each saturation value of the displacing fluid moves at constant speed along the porous media.
- ✓ A displacing fluid saturation moving at higher speed than the rest is developed as consequence of the above. This saturation is called "*Front Saturation*."

And, using Darcy´s formulation, expanded to multiphase scenarios through relative permeability curves, a very simple relationship is obtained between the variables of interest.

$$Swm - Sw_2 = Q_i\,(1 - fw_2) \quad \text{Eq. IV-1}$$

$$Q_i = \frac{1}{\left(\dfrac{d\,fw}{d\,Sw}\right)_{Sw_2}} \quad \text{Eq. IV-2}$$

Where

- ✓ Q_i = Cumulative volume of injected fluid
- ✓ Swm = Mean (average) water saturation
- ✓ Sw_2 = Water saturation at the output face
- ✓ fw = Water fractional flow for any Sw point value

- ✓ fw_2 = Water fractional flow for output face saturation

  ***Note**: The fractional flow curve (fw) must only be a function of water saturation (Sw) to derive the above equations. This is so because the action of capillary and gravity forces is eliminated during the development.*

For a homogeneous, one-dimensional porous medium, these equations relate the mean saturation in unsteady-state flow, the point saturation at output face, cumulative injected volume and fractional flow of flowing phases.

Properties of Fractional Flow Curves

In homogeneous, horizontal, one dimensional systems, dominated by viscous displacement forces, the fractional flow curve obeys the following equation:

$$fw = \frac{1}{1+\frac{Ko\ \mu_w}{Kw\ \mu_o}} \qquad \text{Eq. IV-3}$$

Fig. IV-1 shows a generic water fractional flow curve and some interesting properties of the curve as well.

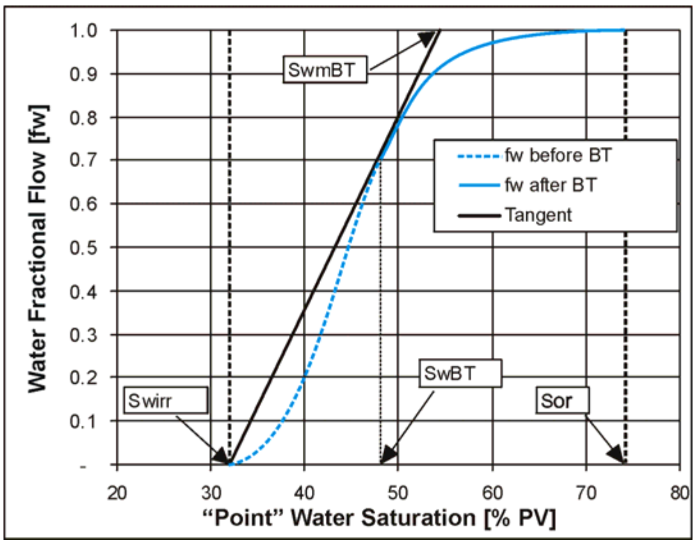

Fig. IV-1 Fractional Flow Curve Properties

By drawing the tangent to fractional flow curve, which goes through Swirr point, the following properties are determined in the figure.

- ✓ Sw at tangency point corresponds to that of the displacement front. It is the water saturation obtained at the output face at "Breakthrough" (BT).
- ✓ Fractional flow, at the same point, corresponds to that obtained at BT.
- ✓ Intersection of the tangent with fw=1 (at the top of the vertical scale) determines mean water saturation in the porous medium, when BT occurs.
- ✓ The dotted section of the fractional flow curve (concavity upwards) determines the range of undefined saturations in the frontal advance theory. It includes all saturations comprised between Swirr and the displacement front saturation.

The lower the mobility ratio between displacing phase and displaced phase is, the higher the value of Sw at the displacement front becomes. When displacing fluid mobility is lower than displaced fluid mobility (M<1), displacement tends to become perfectly "piston-like". In other words, the fractional flow curve is almost fully concave upwards and water saturation at the displacement front approaches the value "100-Sor".

Simplification of Complex Systems

Many authors propose using relative permeability pseudo functions with the purpose of using the simple equations associated to fractional flow curves, when describing more complex systems,.

Briefly, the technique of using relative permeability pseudo functions consists in finding a set of curves that applied to a one dimensional system produce the same results as those obtained in a complex model. Once the more complex systems are reduced to “equivalent” one-dimensional models, the fractional flow equations are applied.

However, this simplification may be inadequate, as shown in the following example.

Homogeneous Block under Predominant Gravity Forces

For this study, a vertical section between injector and producer is selected from a horizontal, homogeneous structure with uniform thickness. This section may be considered as a two-dimensional block, since horizontal thickness is negligible in comparison to its height and length.

In this block, the scenario to be analyzed is that of gravity-dominated water-oil displacement.

The overall properties assigned to the block are:

- ✓ Swirr = 30%
- ✓ Sor = 20%
- ✓ Kro[Swirr] = 0.8
- ✓ Krw[Sor] = 0.3

Under these conditions, the set of relative permeability pseudo-curves adequate for this system takes the shape indicated in Fig. IV-2. These types of relative permeability curves are inherent to horizontal, homogeneous or randomly heterogeneous structures, under predominance of gravitational effects.

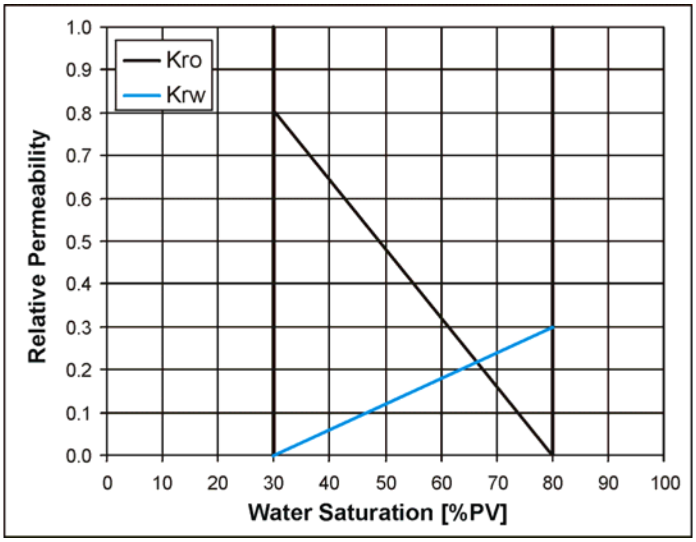

Fig. IV-2. Relative permeability pseudo-curve of a horizontal and homogeneous block under predominant gravity forces.

The curves shown in Fig. IV-2 indicate that, between extreme saturations, the capacity to conduct water or oil is directly proportional to the saturation of each phase (this topic was already analyzed while developing the simple model in Chapter I). These curves are called pseudo-functions because they are not obtained at laboratories (viscous-dominated curves), but using a simple calculation during oil displacement.

Fig. IV-3 shows a very simple layout of oil displacement by water under the mentioned conditions.

In this figure, the displacement process is represented in several steps, each one divided into two stages:

- ✓ During the first stage of each step, water admission takes place under the influence of viscous forces. These stages are plotted in the left column of Fig. IV-3.
- ✓ During the second stage, fluids are redistributed as a consequence of the strong gravity action. These stages are plotted in the right column of Fig. IV-3.

The division into stages is done solely with didactic purposes. In practice, continuous water injection is accompanied by production at the output end and by fluid redistribution. The latter causes an increase in the water level as displacement increases.

***Note**: Fig. IV-3 shows a fluid distribution layout fully dominated by gravity forces. In real displacements, water-oil contact increases more or less horizontally, as a function of the balance of forces during displacement. The qualitative conclusions drawn from this example are independent of the degree of predominance of gravity forces; thus, an extreme case was chosen, to simplify the development.*

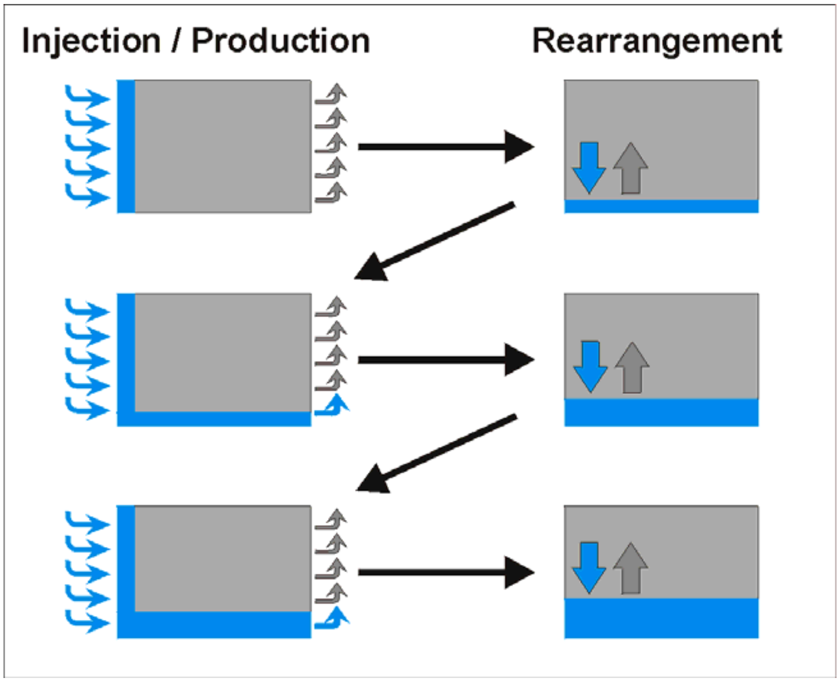

Fig. IV-3: Layout of the displacement in a horizontal homogeneous block, under the influence of gravity forces. For didactical purposes, the process is divided in an injection (and production) stage and a fluid redistribution stage within the structure.

Making the routine calculations (Eq, IV-3), with the relative permeability curves of Fig. IV-2, the system's fractional flow curve can be built for different oil and water mobilities. The result is shown in Fig. IV-4.

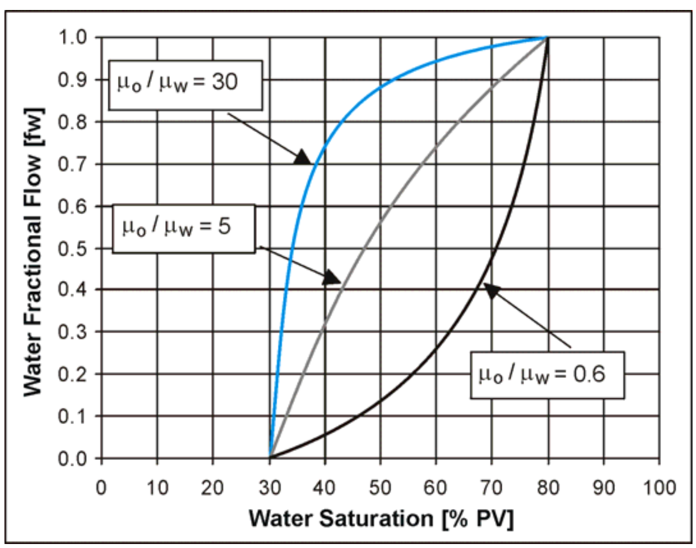

Fig. IV-4: Fractional flow curves corresponding to the relative permeability curves in Fig. IV-2 using different viscosity ratios.

Fig. IV-4 shows that a viscosity ratio of "0.6" (μ_o / μ_w = 0.6) produces a fractional flow curve with concavity "upwards" in the whole saturation range between Swirr and 100-Sor.

Based on the already analyzed properties of the fractional flow curve, the following conclusion could be drawn:

Displacement becomes "piston-like" when viscosity ratio gets down to "0.6", since the fractional flow curve is not valid in the whole range of mobile saturations.

> ***Note**: With a similar example, L. Dake draws the same conclusion in pages 393 and 394 of his book "The Practice of Reservoir Engineering", Elsevier - 1994*

This straightforward conclusion from the frontal displacement theory, where the properties of the fractional flow curve were obtained, conflicts with the example analyzed in this development. Let us remember that the relative permeability curve of Fig. IV-2 was obtained by accepting that gravity forces dominated fluid distribution in the block. This assumption leads to filling up the block with water progressively displacing oil from bottom to top, regardless of oil viscosity.

Besides, a "piston-like" displacement, as suggested by the fractional flow curve, would indicate that BT would occur only after fully replacement of oil by water. And both the relative permeability curve and the "step by step" model indicate that water production occurs from the very beginning of injection.

To clarify the physical meaning of this example, we can summarize it as follows:

1. The proposed model is very simple, comprising a horizontal homogeneous block where water displaces oil under the influence of gravity forces.
2. In this model, the water-oil interface moves like an ascending horizontal (or almost horizontal) plane, from the base to the top of the block.
3. The relative permeability pseudo-function can be easily obtained as shown in Fig IV-2.
4. The fractional flow curves associated to such relative permeability curve "show" that, for certain viscosity ratios, "piston-like" displacement is achieved. The water-oil interface moving as a vertical plane, from input face to output face.

The existing contradiction between items 2 and 4 is more noticeable when taking into account that "piston-like" displacement (vertical interface) is predicted for very low oil viscosities, where gravity forces (horizontal interface) are expected to be more effective.

Summarizing even further the example it may be said that:

1. A displacement model is proposed where the water-oil front moves according to the layout in Fig. IV-5.
2. The "demonstration" concludes that the front moves according to figure IV-6.

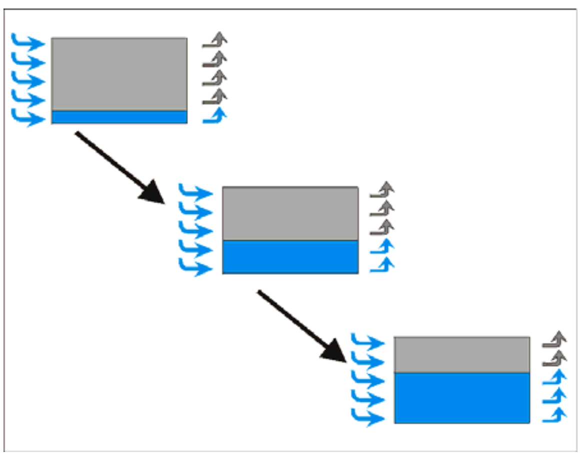

Fig. IV-5: Proposed filling model under the influence of gravity forces. Sw is the same in the whole block during the entire process.

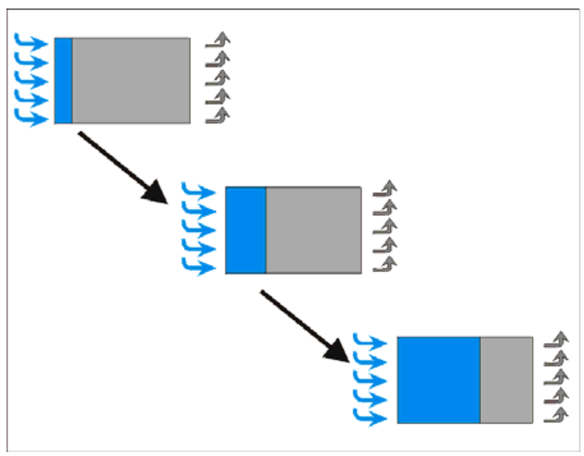

Fig. IV-6: Contradictory result after applying the fractional flow curve properties.

The absurd result indicates that some mistake has been made in the theoretical development, and although the matter will be fully discussed at the end of this chapter, it is easy to infer that the mistake consists in supposing that the properties of the fractional flow curve, obtained in the absence of capillary and gravity forces, can be used to describe gravity-dominated systems.

... Even when simplified to one-dimensional models!

SECOND EXAMPLE
The same relative permeability and two different production histories

In this example two different displacement scenarios are analyzed on the same multilayer layout of heterogeneous reservoir:

- ✓ Non communicated layers (no cross-flow). Predominantly viscous displacement.
- ✓ Fully communicated layers. Fluid distribution dominated by gravity forces.

In order to work with this model, the previously mentioned procedure to simplify complex systems is used, through the generation of relative permeability pseudo-curves applicable to an equivalent homogeneous reservoir. In other words, by means of an adequate procedure, relative permeability curves are generated allowing the operation with the 10 layer block schematized in Fig.lV-7 as if it was a horizontal block composed of a single homogeneous layer equal in thickness to the sum of the 10 layers individual thickness.

In the ideal case, the so-called relative permeability pseudo-curves should lead us to the same response –from a homogeneous block- as that obtained by studying the 10-layer system with independent properties for each of them.

The simpler methodology (possibly the most known one) for the generation of relative permeability pseudo-curves is detailed in L. Dake "The Practice of Reservoir Engineering", Elsevier – 1994, on pages 370 and following.

The proposed methodology for an N-layer system consists in determining a flooding order to these layers, and assign them a correlative index (between 1 and N) allowing for the resolution of the following set of equations during water injection.

$$Sw_n^* = \frac{\sum_{i=1}^{n} (1 - Sor_i)\, \phi_i\, h_i + \sum_{i=n+1}^{N} Swirr_i\, \phi_i\, h_i}{\sum_{i=1}^{N} \phi_i\, h_i} \quad \text{Eq. IV-4}$$

$$Krw_n^* = \frac{\sum_{i=1}^{n} Krw[Sor]_i K_i\, h_i}{\sum_{i=1}^{N} K_i\, h_i} \quad \text{Eq. IV-5}$$

$$Kro_n^* = \frac{\sum_{i=n+1}^{N} Kro[Swirr]_i K_i\, h_i}{\sum_{i=1}^{N} K_i\, h_i} \quad \text{Eq. IV-6}$$

Subindex "i" increases as water reaches the output face in a new layer. When the "n" layer is filled, Sw values in the output face (Sw_n^*) and the corresponding water relative permeability values (Krw_n^*) and oil relative permeability values (Kro_n^*) are obtained by solving equations IV-4, IV-5 and IV-6.

According to this methodology, calculations are performed using permeability end points (Krw[Sor] and Kro[Swirr]) for each layer. The basic assumption used to build this set of equations is that each layer is solely at one of the two saturation end points (Swirr or Sor). This assumption is reasonably adequate for mobility ratios equal to or lower than "1". In other words, displacement is assumed to be "piston-like" in each layer, so that, at BT time, conditions change from Swirr to Sor in the output face of the waterflooded layer.

> ***Note**: Due to the unavoidable heterogeneity of real porous media, "perfect piston-like" displacements do not occur in nature. However, some "piston-like" displacements are used in this book to make it easier to obtain numerical values. Although these displacements model an extreme situation, the conclusions that may be drawn do not vary qualitatively when displacement is somewhat less efficient.*

The set of resulting values (Sw*, Krw* y Kro*) is known as relative permeability pseudo-curves. As stated above, the basic assumption behind its construction is that these curves, applied to the set of layers as if they were a single homogeneous block, produce the same results as those obtained when considering an N-layer system, with relative permeability curves for each layer.

The reservoir scheme, comprising 10 layers of equal thickness and porosity with permeabilities decreasing from bottom to top of the structure can be seen in Fig. IV-7. Permeabilities have arbitrarily been set between 100 mD and 1,000 mD.

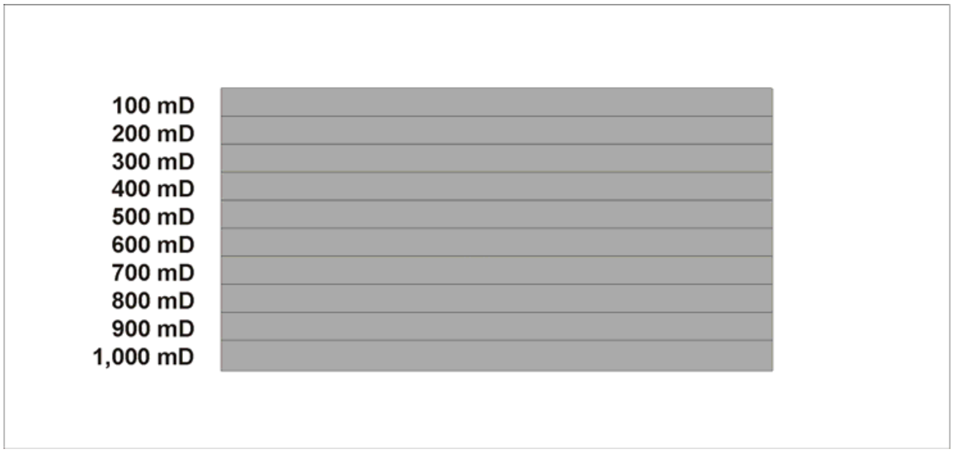

Fig. IV-7 . Reservoir scheme with 10 parallel and superimposed layers.

The proposed scheme is very simple but the conclusions to be drawn are valid for any more complex model. The use of a simple model makes it possible to focus the analysis on the essential concepts of fluid displacement.

The permeability of the set of parallel layers is determined by the equation:

$$K_{Average} = \frac{\sum_{i=1}^{10} K_i \times h_i}{\sum_{i=1}^{10} h_i} \quad \text{......... Eq. IV-7}$$

By solving Eq. IV-7 for the model, the absolute permeability value of the 10 layers is obtained:

$K_{Average}$ = 550 mD

For reasons that will be apparent at a later stage (related to the possibility of using the relative permeability concept), oil displacement with water is analyzed using a mobility ratio ("**M**") equal to "1".

$\mathbf{M = (K_w/\mu_w)/(K_o/\mu_o) = 1}$

As already stated, this characteristic allows displacement to be considered as very efficient (almost "perfect piston-like") within each layer, assuming each layer is intrinsically homogeneous.

Case I: Non-communicated Layers (no cross-flow).

In this case, each layer is swept at a speed directly proportional to its permeability. As mobilities are equal (M=1), once the displacement pressure is established, replacement of one phase by another does not alter the overall fluid mobility. As a result, during displacement, the total production rate of each layer remains constant. Thus, the 1,000 mD layer is swept 10 times faster than the 100 mD layer.

All layers are assigned the following values:

- ✓ Swirr = 25 %
- ✓ Sor = 30 %
- ✓ Kro[Swirr] = 1.00
- ✓ Krw[Sor] = 0.33
- ✓ μo = 3.00 cp
- ✓ μw = 1.00 cp
- ✓ Displacement = Very efficient ("perfect piston-like" displacement is assumed within each layer).

When Breakthrough (BT) occurs in the most permeable layer (when the water front reaches the production face), only 10% of the length of the less permeable layer has been swept, and the other layers have been swept proportionally to their permeability. This situation is shown in Fig. IV-8.

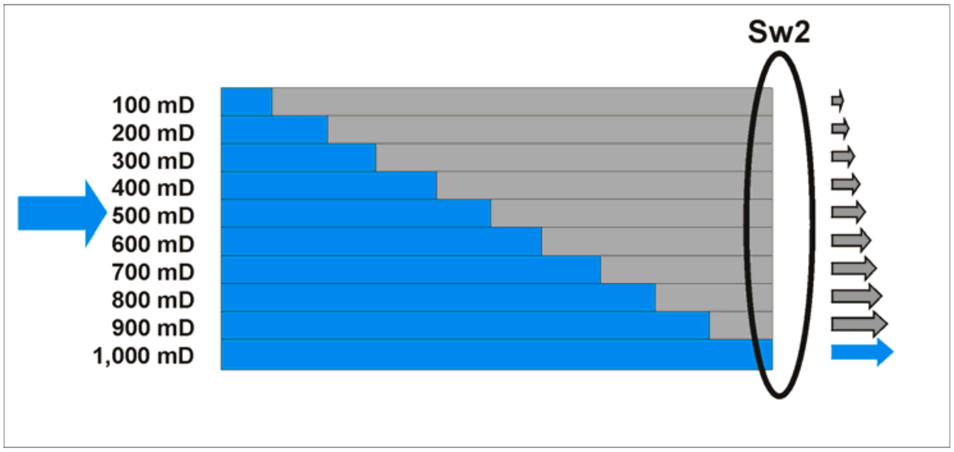

Fig. IV-8: Non-communicated layers. Water saturation at the BT of the more permeable layer

Applying Eq. IV-4, Eq. IV-5 and Eq. IV-6 as displacement evolves, the relative permeability pseudo-curve is obtained, as plotted in Fig. IV-9.

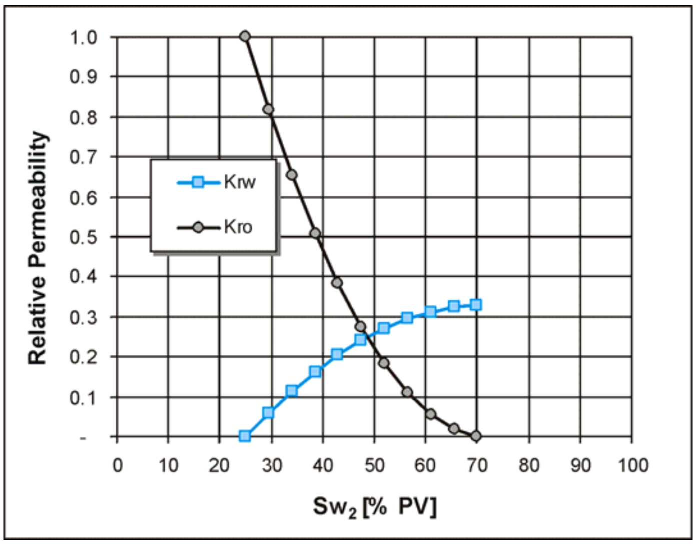

Fig. IV-9: Relative permeability pseudo-curve for the system in Fig. IV-8.

The corresponding values are shown in Table IV-1.

Table IV – 1

Relative Permeability Pseudo-Curve

	Sw_2 [% PV]	Kro	Krw
1	25.00	1.000	0.000
2	29.50	0.818	0.060
3	34.00	0.655	0.114
4	38.50	0.509	0.162
5	43.00	0.382	0.204
6	47.50	0.273	0.240
7	52.00	0.182	0.270
8	56.50	0.109	0.294
9	61.00	0.055	0.312
10	65.50	0.018	0.324
11	70.00	0.000	0.330

Case II: Interconnected Layers. Full Gravitational Segregation.

In this case, layers are filled from bottom to top of the structure as injected water reaches the gravitational balance with oil within the porous structure. Displacement progresses following a similar layout to that shown on Fig. IV-3.

Thus, the 1,000 mD layer is swept first, simply because it is in the low part of the structure. Fig. IV-10 plots the situation when the bottom layer becomes fully waterflooded.

The vertical arrows drawn inside the block indicate the direction of fluid redistribution while displacement progresses. This "internal" movement does not lead to fluid production but to fluid redistribution as a result of their different density.

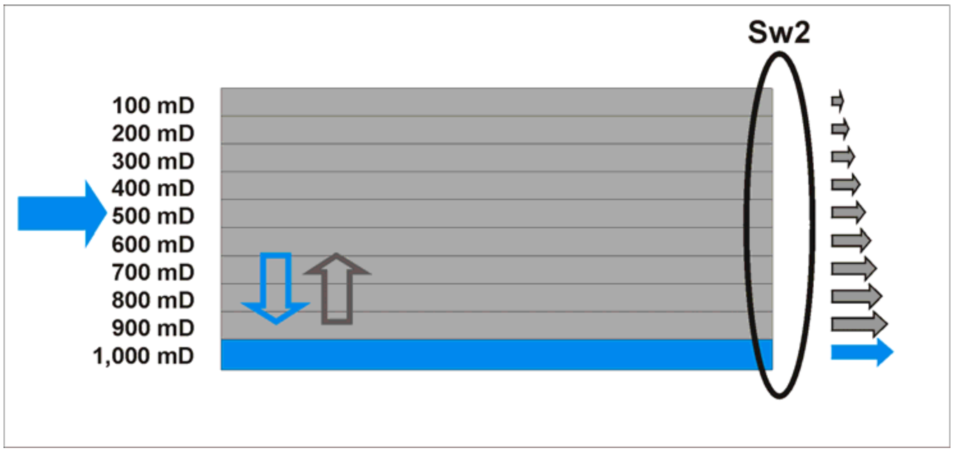

Fig. IV-10: Distribution of fluids under gravitational predominance. Water saturation when sweeping of the most permeable layer is completed.

By solving Eq. IV-4 through Eq. IV-6, as previously, the pseudo-function obtained is **exactly** the same as in Case I, since the layers are filled in the same sequence. The situation is surprising because, although the relative permeability curves are equal, the production histories are entirely different. Comparing Fig. IV-8 and Fig. IV-10, it can be seen that the same water-oil ratio (WOR) occurs in both cases, at the time of the plot, when the more permeable layer is producing water and the remaining layers are producing oil. Nevertheless, the cumulative oil production is much higher in the scenario plotted in Fig. IV-8 than in the scenario plotted in Fig. IV-10.

The water-cut curve as a function of produced oil for the first case (non-communicated layers) and for the second case (fluid distribution under gravitational predominant forces) are shown superimposed in Fig. IV-11.

In this figure it is possible to appreciate the significant difference in production histories of each model, in spite of both having the same relative permeability pseudo-function.

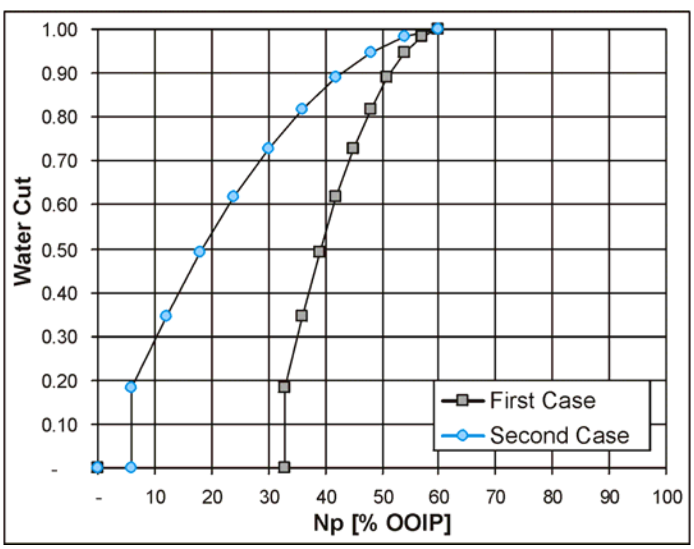

Fig. IV-11: Water cut curve and oil production ratio for the two cases studied.

Further Explanations

Both models in the same reservoir

For the first case it was assumed that layers are not communicated. This restriction is not fully necessary with M=1, since under these conditions pressure gradients in all layers are identical (linear); therefore, viscous forces do not contribute to cross flow.

Based on the above explanation, it can be seen that both cases may occur in the same reservoir. With high rates, the system would approach the first case, and with very low rates, the action of gravity leads to the situation presented in the second scenario.

Minor differences

From a technical point of view, pseudo-functions are slightly different for the two cases in the previous discussion. In the first case the function is stepped, while in the second case the points are joined by continuous straight lines according to the filling scheme. However, the division into 10 discrete layers is arbitrary, since as many layers can be selected as necessary to define a continuous function, or simply a continual gradation of permeabilities ("infinite" layers) with identical result.

Why M=1?

In the example plotted in Fig. IV-8 (non-communicated layers), Darcy's law is used to calculate relative permeability pseudo-functions, by relating rates to pressure gradients. This is made possible only because the pressure gradient is the same in all layers, in addition to being the same for the entire length of the model. As soon as M ceases to be 1, gradients in each layer suffer a sudden rupture in the interface (or a continual variation), different for each layer. In such case, the calculation of the relative permeability curve at the output face is conceptually incongruent since, although there are perfectly defined production rates, there is not a unique pressure gradient (each layer has its own gradient). Besides, under these conditions, the ratio between gradients of the different layers varies with time. And Darcy´s law, for the whole system, requires a unique gradient at each point of the calculation.

Consequently, the relative permeability curves obtained for the situation shown in Fig. IV-8 for M=1 not only becomes invalid for other M values, but also lacks physical meaning, because Darcy's law is not definable at the output face.

Generalization

The previous analysis show a very simple case showing that the same pseudo relative permeability curve may apply to two very different production histories.

This case is not an exception. On the contrary, it is the general rule for all cases where not only viscous forces are influencing fluid distribution during displacement.

Now… we should ask a simple question:

How can we determine whether only viscous forces are acting in the reservoir?

The answer is "simple" too: Whenever production ratio (water-oil or gas-oil) depends on flow rates, it strongly implies that other than viscous forces are acting on the production mechanisms. The frontal displacement theory (characteristic of viscous displacement), is the only theory showing independence between the ratio of fluids production and overall flow rate.

FAQs

Question: The used methodology to generate pseudo-functions is very simple and not all the authors agree upon its use. Are the problems analyzed in this chapter also found with dynamic pseudo-functions?

Answer: Problems are also found with dynamic pseudo-functions when fluid distribution is influenced by capillary or gravity forces. On this chapter the analysis has been done with a very simple (widely known) methodology to facilitate the understanding of the examples.

For the time being it is sufficient to mention that all methodologies to obtain pseudo-functions lead to curves of the type presented in Fig. IV-2 for homogeneous, horizontal blocks, when gravitational forces dominate fluids distribution. These curves, when fed into a numerical simulator, result in production forecasts very far from reality.

Question: How should relative permeability pseudo functions be generated?

Answer: As demonstrated in previous chapters, relative permeability curves (either direct or pseudo functions) are never adequate to reproduce **production** histories. Curves representing what we call Specific Productivity (**SPC**) are needed, and their construction is discussed in the following chapters. Anyway, increasing (not decreasing) the vertical grid density in numerical simulators is the best way to adequately evaluate gravity influence on fluids distribution. Summing up, pseudo functions should be applied to minimize the horizontal gridding using specific productivity curves (SPC) without reducing the number of layers of the vertical grid.

Comment: Developments presented so far only show some deficiencies in the traditional use of relative permeability curves. But very little improvements may be obtained from the above mentioned concepts using available calculation tools.

Answer: The comment is correct to a great extent. However, the main purpose of this book is to explain why recurrent problems are met with the use of relative permeability curves to describe production in oil and gas reservoirs. Solutions in this book are mainly conceptual. The operative tools will only be developed as long as material balances and numerical simulators include the variables really representative for production in hydrocarbon reservoirs description. As already mentioned, such variables are average saturation and fluid specific productivity (or injectivity).

Question: Is it possible to develop numerical simulators including these concepts?

Answer: Certainly. However, this topic will be discussed in following chapters.

SPECIAL COMMENTS

Although concepts presented below could be included in the FAQs section, it's significance deserves a separate treatment.

Gravity and Capillary Forces in Darcy's Equation

In multiphase flow, Darcy's equation can be written, for each phase, taking into account capillary pressures of the system and gravity contribution. Consequently, some questions arise, namely:

Why, as stated on this book, relative permeabilities only have physical significance when capillary and gravity forces are absent?

In fact, this is one of the questions more frequently asked to the author, concerning the developments presented in this work.

The answer is that these forces may exert influence over the fluids in the porous medium through two different phenomena:

- ✓ Moving fluids toward or from the porous medium.
- ✓ Redistributing fluids inside the porous medium.

When fluids are displaced, both forces contribute to its movement in the same way as an external force would do (viscous forces). Darcy's equation, modified by relative permeability curves for multiphase flow description, covers this situation.

In redistribution, capillary and gravity forces produce a change in fluid saturation at different points in the porous structure. This situation is not covered in any finite system by Darcy's equation once modified for multiphase flow. This is so because:

- ✓ During fluid redistribution, each fluid partially occupies the space previously occupied by another fluid. To do so, some back-flow phenomena occur. Namely, while one fluid moves in a certain direction, the other one moves in the opposite direction.
- ✓ Proportionality between applied potential difference and production flow rate is lost for each phase.

Fig. IV-3 shows the mentioned back-flow illustrating the reason of such loss of proportionality. The capacity to produce water and oil through the output face is different before and after phase redistribution occurs. In other words, the same average saturation originates different production proportions of fluids

This phenomenon occurs whenever the phases are redistributed inside the porous medium.

Redistribution degree depends on the balance reached among the acting forces. The lesser the action of viscous forces, the greater the phase redistribution derived from capillary and gravity phenomena.

In practice, the production rate is the main external factor influencing the forces balance.

Then,… as a consequence, relative permeability curves not depending on flowing rates can only be defined when there is no fluid redistribution due to capillary or gravity causes.

> ***Note:*** *Comments in this section apply to porous media with finite volume. In the event of point "porous media" (without volume), the relative permeability concept definition could be held when capillary and gravity effects are present. In this case, fluid redistribution is impossible, because there is no volume where it can take place. But is it is also rather abstract to talk about fluid saturation or capillary pressures in a system without volume.*
>
> *Any practical application of the relative permeability concept involves discrete volumes of porous media. With finite volumes, redistribution phenomena are unavoidable if capillary and/or gravity forces are present.*

The Fractional Flow Curve and Capillary Forces

In this chapter, we clearly state that there is a range of saturation values (between Swirr and SwBT) in which the fractional flow curve is not defined. However, a great number of publications assign physical existence to saturation points in that range, based on capillary phenomena.

This reasoning is supported on the following premises:

- ✓ To draw a saturation profile according to the displacement theory, capillary effects are neglected.
- ✓ The developments of that theory lead to "apparent" multiple saturations for a given point, precisely in the saturation range where the fractional flow curve concavity is upwards.
- ✓ If capillary phenomena were taken into account, such multiple saturation values would disappear, and the fractional flow curve would have physical sense in the whole saturation range.

However, this type of reasoning is not consistent.

Perhaps the easiest way to prove the error in that line of discussion is by reduction to the absurd.

To develop the frontal advance theory it is not necessary to mathematically "neglect" the effect of capillary phenomena. These phenomena can be physically eliminated, by means of ultra-low tension surfactants, or with porous media showing no preferential wettability to any fluid (90º contact angle at the rock-fluid interface).

By choosing the second alternative, as the contact angle does not appear in the development of the frontal advance theory, the whole development becomes valid if contact angle is 90º.

And for a 90º contact angle, it is not possible to resort to capillary phenomena to "justify" the upwards concavity area in the fractional flow curve, simply because there are no capillary pressures.

Therefore, the explanation should necessarily be different.

The explanation lies in that the fractional flow curve is built as a function of point saturations, not as a function of the system's average saturations. When a saturation front is present, it is inevitable to have not defined (inexistent) point saturations in the system.

Average and Point Saturations

In the two previous analyses, it is frequently mentioned the difference between point and average water saturation values. This situation appears frequently when relative permeability curves (or the associated fractional flow curve) are used to describe real displacement phenomena.

Any physical displacement phenomenon involves finite volumes. However, on account of the reasons widely discussed in Chapter III, in unsteady-state systems, relative permeability curves are calculated at one point (the output face of a one-dimensional system). And properties at one point are not necessarily useful to describe average properties. In fact, some properties are qualitatively different in each case.

Continuous and Discontinuous Values

While average saturations may adopt any value between 0 and 100% for each fluid, point saturations may have discontinuities (not defined values).

The conceptual difference between average and point saturations can be visualized in a glass of water. At any point in the glass, water saturation (Sw) may only have one of two possible values: Sw is 0 % at any point above the water-air interface, and 100 % below such interface. In other words, there are no water saturation points with intermediate values (e.g.: Sw = 15.5%). However, mean saturation of the glass (or of any finite volume in the glass) may have any intermediate value between such extreme values.

The same phenomenon occurs in porous media where a displacement front exists. This front originates a saturation leap where point saturations are not defined..

Summing up, the repeated effort in specialized literature to find a physical meaning of fluid saturations between Swirr and displacement front saturation is due to the fact that the curve is built with point values (where the inexistence of a range of values is perfectly acceptable), and it is used to describe real systems, on account of its average saturation (where all saturation values are needed)

Volumetric Properties

Some physical properties or phenomena may only occur in three-dimensional systems. An unequivocal example is the above mentioned fluid redistribution. In dimensionless (point) systems the concept of fluid redistribution does not make any sense.

In all real and discrete systems volume is not null.

Summing up, by using point properties (such as relative permeability curves in unsteady-state systems) to describe real systems, incompatibilities or inconsistencies such as those described in this chapter, will show up sooner or later.

The correct way to describe finite volume systems is through average properties, as explained in this work.

SUMMARY AND CONCLUSIONS

Briefly, the problem posed in the two examples of this chapter (and in a publication[5] by Virués, Tellería and Crotti), originates on the following:

1. The frontal advance theory, developed by Buckley and Leverett[3] and completed by Welge's[4] work, leads to a simple description of immiscible displacements, in one-dimensional, homogeneous porous media, under viscous force influence.
2. It is a usual practice to simplify complex systems reducing them to one-dimensional systems so to obtain a unique relative permeability curve (pseudo-function or similar construction). Once the more complex model is simplified, the available simple equations are applied to describe its behavior.

However, the fact that the second point is only valid if it does not conflict with the first point is usually not considered. And point 2 conflicts with point 1 every time we face systems, either heterogeneous or with fluid distribution influenced by forces other than viscous.

In the examples analyzed, pseudo-functions built under the assumption of absolutely dominant gravity forces are used. It is not surprising that, on applying the analyses developed for viscous-dominated models, results obtained are apparently paradoxical.

This problem is much more generalized than the one posed in this chapter. In fact, any time a complex model is simplified and brought into a one-dimensional, homogenous model, some initial assumptions are violated thus leading to erroneous interpretation of results.

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