Chapter III

> *"From a physical point of view strictly steady-state conditions of heterogeneous-fluid flow in oil-producing systems are virtually never encountered. The mechanism of oil production is inherently one of a continual change in the volumetric contents of the oil-producing section. Of necessity, the oil expelled must be displaced either by gas or by water. Accordingly, as production proceeds, the average oil saturation in the original reservoir monotonically decreases--except when the production is the result of reservoir liquid expansion-while the resultant saturation of the displacing phase of gas or water, or both, simultaneously increases."*
>
> *M. Muskat. "Physical Principles of Oil Production", 1949 .*

INJECTION, CONDUCTION AND PRODUCTION

The introductory quote is closely related to the conceptual discussion in this chapter. This will become evident when analyzing the concept of relative permeability further on in this chapter.

For the time being, it is important to mention that the man considered to be the "father" of reservoir engineering (M. Muskat) has introduced the following words and expressions in a single paragraph:

- ✓ It is virtually impossible to encounter systems in steady-state conditions under multiphase displacement.
- ✓ Four times, Muskat uses words related to oil production in real oil-producing reservoirs, but he does not mention fluid conduction.
- ✓ Besides, he refers to the average fluid saturation as a variable of interest.

It is worth noticing that concepts presently used to describe fluid movement in reservoirs were being developed at the time the above-mentioned paragraph was written. This is a significant remark, because critical analysis of such concepts is done in this chapter beginning, naturally, with Darcy's law.

This law was already introduced in chapter I of this book. For homogeneous and horizontal linear systems Darcy's law equation is shown below:

$$\mathbf{Q = K\ A\ \Delta P / (\mu\ L)} \quad \text{.......... Eq. III-1}$$

This equation relates fluid flow rates, in a given porous media, to fluid geometry (area and length), to fluid properties (viscosity) and to operating conditions (pressure difference across the media).

The parameter relating the above mentioned variables is known as "permeability" (**K**). Permeability is a property of a porous material, not dependent on fluid geometry, flow, or operating conditions.

A definition was given for "permeability" in Chapter I:

"Permeability is the capacity of a porous medium to conduct fluids"

It was also mentioned that, in practice, such **conduction** capacity is determined by direct measurement of either incoming or outcoming rates. In other words, the model used for permeability measurement and calculation in porous media, assumes that the capacity to **conduct** a fluid is equal to the capacity to **inject** or **produce** such fluid.

This model (**Conduction** = **Injection** = **Production**) is absolutely valid in the condition where Darcy's Law was defined: Incompressible and linear fluid flow in a monophasic system

MULTIPHASE FLOW

Relative Permeabilities

As already stated by M. Muskat, monophasic flow conditions are practically never encountered in real reservoirs. On the contrary, biphasic or three-phase flow is frequent. In these cases, according to practices adopted by our industry, Darcy´s equation is still used but with the addition of a correction factor. This correction factor is known as relative permeability and it's value is only dependent upon the system's fluid saturation.

In these cases, taking as example simultaneous water and oil flow, Eq. III-1 becomes:

$$\mathbf{Qw = K\ Krw\ A\ \Delta Pw / (\mu w\ L)} \quad \text{.......... Eq. III-2}$$

$$Q_o = K\,K_{ro}\,A\,\Delta P_o / (\mu_o\,L) \quad \text{Eq. III-3}$$

Where the only changes to Eq. III-1 are:

- ✓ The mentioned correction factor: Krw for water flow and Kro for oil flow.
- ✓ Subscripts ("o" for oil and "w" for water) to identify the phase being characterized.

This generalization will prove to be right only if each phase flow is proportional to the pressure difference applied to such phase, for a given phase saturation.

***Note:** If there is no capillary pressure, both pressure differences are coincident.*

The expressions "**K Krw** " and "**K Kro**" are usually replaced by "**Kw**" (effective permeability to water) and "**Ko**" (effective permeability to oil). "Effective" means that both "**Kw**" and "**Ko**" perform the function previously performed by absolute permeability (**K)** in Eq. III-1.

However, the fact (not mentioned in textbooks nor in technical publications) that in unsteady systems the equivalence among the three actions (**conduction, injection** and **production**) is broken, is often overlooked when making this generalization of Darcy's law.

As we will see this fact is very far from being trivial.

The entire structure supporting the regular use of relative permeability curves is affected as a direct consequence of the above mentioned fact.

Summing up, in multiphase flows, the use of Darcy's law is extended by defining effective permeability to a phase as follows:

Effective permeability to a given fluid phase is the capacity of a porous medium to conduct this phase at a given fluids saturation.

Injection, Conduction and Production

In general, once assumed that the system is in unsteady-state condition, such **conduction** capacity cannot be evaluated by measuring neither the injection capacity nor the **production** capacity.

At this point, it is advisable to remember that in real cases (description of dynamic behavior of complex reservoirs) the data of interest are:

- ✓ Average saturation of each studied block (the entire reservoir on a material balance or a cell in a numerical simulator).
- ✓ Injection capacity at a specific point in the block (injection well or contact with nearby cells)
- ✓ Production capacity at a specific point (producing well or contact with nearby cells).

Thus, the capacity **to conduct** a certain fluid becomes a rather abstract concept from the point of view of the oil industry, since interest is focused on the capacity **to inject** or **to produce** such fluid.

When a system goes through a transient state (as it happens in real reservoirs during exploitation), a given fluid ratio can be *injected*, another one can be *conducted*, and a fully different third one can be *produced*.

In the previous chapter, the way in which the difference between **conduction** and **production** affects measurement and transference of lab information to reservoir was shown. The example analyzed was that of a routine lab measurement. However, as it will be fully demonstrated, the same concepts are applicable to all cases of multiphase flow in porous media.

Validity of the Relative Permeability Concept

A direct consequence of relative permeabilities being a correction factor in Darcy's equation is that said curves lose their physical meaning if, for any reason, Darcy's law becomes not applicable. This fact is not trivial, as we will see.

In technical literature, a good deal of cases where Darcy's law is not applicable could be found, mainly on account of linearity loss between the pressure gradient and the system's flow rate. Although this situation represents an obvious case were relative permeability curves become meaningless, in the following development we analyze a global situation which affects in a more fundamental way the ordinary use of relative permeability curves.

The thesis to be analyzed may be summarized as follows:

Many times, Darcy's law is not applicable simply because the equation's variables are not defined.

At this point it is important to remark that the phrase "...are not defined" does not refer to a particular measurement or methodology problem. It means that it is impossible to assign values to such variables, and, if values are assigned (which is frequently done), the results obtained from the application of Darcy's equation are no longer valid.

Undefined Variables

The definition of permeability (the core of Darcy's equation) implies determining the **conduction** capacity for a certain fluid. On the other hand, in a real unsteady case, the only possible measurements are the **injection** or **production** capacities for such fluid.

The following analogy, related to a discipline quite different from that of fluid movement in reservoirs, will contribute to adequately focus on the problem.

Let us suppose that we attempt to describe the properties of an object by answering a number of questions in a form, with specific boxes, that allow for only one word or data for every item. During the process we may encounter some difficulties such as those described in the following paragraphs.

Surely, we may answer with no ambiguities the question *"Weight of the object?"* by filling the box with the pertinent weight in Kg.

The same is true for other properties of the object, such as length, width, etc...

But let us suppose we are asked for the color of the object, and we are allowed to choose among 20 pre-defined colors. Let us additionally suppose that the object is a half-yellow, half-blue basketball ball.

A bit astonished (and bothered) we revise the list of colors and find three alternatives:

- ✓ Yellow.
- ✓ Blue.
- ✓ Green.

Yellow or **blue** are bad descriptions of the object, but **green** (the result of mixing 50% yellow and 50% blue) appears to be worse. For instance, if someone is asked to pick-up the green object in the room, it will be difficult for him to identify the blue and yellow ball with the requested object.

As shown below, this funny example is not so far from the real situation that we found when attempting to turn monophasic flow equations (a single color) into multiphase flow (several colors). If fluids (colors) are uniformly distributed, it is valid to say "green" to describe the situation, but if fluids (colors) are not uniformly distributed, a severe difficulty is encountered.

In brief, the previous analysis leads to following conclusions:

- ✓ If we are given one single color to describe an object, we will be able to do it correctly if pigments are uniformly distributed. In other words, the object can be adequately described only if it is monochromatic.
- ✓ If we are given a single box to describe the fluid conduction capacity in a multiphase system, we will only be able to do it adequately if fluids are uniformly distributed. In other words, we will only be able to reach our goal if fluid saturation is only one.

Going back to the use of relative permeability curves as a correction factor of Darcy's equation, to apply such equation to multiphase flow, fluid saturation must be perfectly established and have a unique value in the system to be described. This is the necessary condition for each fluid **conduction** capacity to be only one. If fluids saturation is variable (finite volume system in unsteady state) the same variability is true for the **conduction** capacity of every one of those fluids.

Historical "Solutions"

In practice, two "solutions" were found to avoid the ambiguity between the definition and the measurement possibility:

Chronologically, the first "solution" was to re-create the original conditions in which Darcy's law is applicable:

Injection = Conduction = Production.

Thus, any one of these properties can be measured to obtain the value of the property we are interested in.

> *Note*: ***Permeability*** *is only a measurement of the* ***capacity to conduct*** *fluids. In the general case, the system's permeability does not indicate its capacity to admit or produce fluids.*

This first "solution" was reached through the steady-state measurement system where a certain proportion of both phases is **injected** until **production** becomes equal to **injection** (steady-state is reached). At this stage, all the variables in Darcy's equation are perfectly defined (the ball is Green since the same proportion of blue pigment and yellow pigment is found everywhere).

The second "solution" was found by creating an object of study where the three actions (**injection**, **conduction** and **production**) are identical **by definition**. To obtain this situation, a null-thickness section is taken in the porous medium (a bidimensional lamina). In a null-thickness lamina, everything **injected** is unavoidably **produced.** As a consequence, the same **"conduction**" could be assumed**.** This condition cannot be achieved in any real measurement, since thickness of real objects is not null. In fact, it is achieved by means of a relatively complex calculation, using the unsteady-state methodology[1,2,3,4] already analyzed in Chapter II. In this case, the average values of the system are measured and calculations are made regarding what would happen in a null-thickness sheet. For practical purposes, such lamina is the system's output face since it is the only place where the value of one of the actions (the **production** of the system) can be physically evaluated.

In homogeneous media, both "solutions" provide the same result, and that solution has been used for more than 60 years by our industry to obtain the relative permeability curves as a correction factor of Darcy's equation.

Analogously, both in Physics and in Mathematics complex problems use to take certain particular solutions where equation resolution becomes quite simple. And, at first sight, this seems to be the methodology chosen to measure relative permeability values in multiphase displacements. However, it is advisable to focus on two aspects of this approach.

- ✓ Particular solutions are not useful to describe general cases.
- ✓ In Darcy's equation, resorting to particular cases to simplify complex equations is not a real simplification but the only way to obtain a defined value for some calculation variables.

And, over the years, it appears not to have been taken into account the fact that in real scenarios the reservoir is almost never "Green" (uniform saturation). On the contrary, as a general rule we find different colors (different fluid saturations) in each part of the system.

The Practical Problem

A simple example will enable to seize the magnitude of the problem:

Let us suppose we have a thin horizontal tube, 1 m long (thinness contributes to prevent gravity from segregating fluids inside). If the tube is empty, it is very easy to show that its capacity to conduct water is null ($Kw = 0$ since $Qw = 0$). In spite of how big the pressure difference applied between both ends of the tube may be, the empty tube cannot conduct water unless there is water inside it. In terms of Darcy's equation, we would say that effective permeability to water is zero when water saturation is zero.

Nevertheless, the capacity to **inject** water in that tube is far from being null although the capacity to **produce** water is certainly zero until water reaches the output end. In other words, water can be **injected** without water being **produced**.

Although this example appears consistent enough to differentiate the three actions (**injection**, **conduction** and **production**) in an unsteady-state system, it is advisable to analyze what happens when the tube has been filled with water up to 50% of its length.

What is the tube's capacity to conduct water when water saturation is only 50% (when the water "front" is half way between the **input face** and the **output face**)?

Before answering hastily, we should take into account that we are back to the case of the blue and yellow ball. We had two colors in the ball, and we were asked to describe it using only one color. Now there exist two well-defined **conduction** capacities (The "X" value resulting from flow equations in the waterflooded area and zero in the zero water saturation area).

Which is the adequate value to use in Darcy's equation?:

- ✓ X ? (Blue?)
- ✓ 0 ? (Yellow ?)
- ✓ X/2 ? (Green?)

When answering this question (which does not appear to be easy), it is necessary to take into account that Darcy's equation requires the **conduction** capacity of the system, having available a single box (**K**), where only one value can be placed. Somehow, under these conditions, Darcy's equation becomes the funny form used in the example of the Basketball ball.

Additionally, only the **injection** or **production** capacities are of interest in any real oil industry scenario. The **conduction** capacity is an abstract entity not quantified by any instrument or methodologies related to reservoir description.

When **injection** or **production** of a certain well are mentioned, it is taken for granted that somehow the fluid is transmitted (**conducted**) to or from the well, but the really quantifiable variables are those from the well itself.

FAQs

Comment: I cannot understand why in an unsteady-state system there is not a perfectly defined conduction capacity at each point.

Answer: In this work, the existence of a perfectly defined conduction capacity for each point is not denied. The above analysis highlitghs that, in unsteady systems, there is no conduction capacity for a finite volume block, however small it may be. It is the same as a landscape photograph: For each point ("pixel", in modern language) there is a perfectly defined color. However, there is no single color to adequately characterize the photograph (or any other finite portion of same).

Comment: But..., the same example suggests that, by adequately discretizing the system (up to the individual "pixel" size), it is possible to define the whole as the sum of its parts.

Answer: This is possible when we are dealing with colors, since it is a static property, identical for individual "pixels" and for the whole. In fluid movement in a porous medium, this is not possible with relative permeabilities because they describe a property (**conduction**) other than the property needed to describe the system (**injection** or **production**). Exaggerating the analogy, however perfect the photograph of a rose can be, it will not be able to transmit the rose's fragrance.

Question: Then, what is the purpose of measuring the conduction capacity if this is not the property of interest?

Answer: This question summarizes the purpose of this book. The reason, as I can trace it, is historical. Relative permeabilities were born as a steady-state concept, where Darcy's equation is essentially valid for multiphase flows. In this scenario, **injection**, **conduction** and **production** are equal. The divergence arises when attempting to apply the "steady-state" solution to "unsteady-state" problems, where **conduction** lacks its physical meaning.

SUMMARY AND CONCLUSIONS

In order to emphasize the purpose of this development, it is advisable to summarize the steps we followed along this chapter:

1. Darcy's equation is very simple and adequate to characterize monophasic flows in porous media.
2. The model adopted to describe multiphasic flows is based on the monophasic flow equation, corrected by a factor in the form of a relative permeability curve.
3. Once this factor has been determined (experimentally or analytically), it only depends on saturation of the different phases in the porous medium.
4. However, the fluid conduction capacity (the core of Darcy's equation) cannot be defined in real systems with non homogeneous saturation. These systems only have perfectly defined **injection** and **production** capacities.

The answer to the problem (how to describe multiphase flow in real systems?) is not the one historically used in our industry. From its origins, our industry has answered …

"Green" !!!.

And this is the main source of the problems arising when trying to describe multiphase flow in porous media. For instance, it is pertinent to remember that Darcy's equation is solved with 'green color' in a numerical simulator cell. Among other things, this leads to the well known phenomenon of numerical dispersion[5].

The solution to the problem lies in the use of equations describing the relationship between the variables really defined in reservoirs. These variables are those historically used to characterize reservoirs.

- ✓ Fluid average saturation of the block under study.
- ✓ Capacity to inject fluids in such block.
- ✓ Capacity to produce fluids from such block.

In the previous chapter it was demonstrated that specific productivity curves (**SPC)** adequately meet this purpose. The case analyzed is a lab sample under viscous fluid. However, there are also solutions for all remaining real situations. The detailed analysis of additional scenarios is delayed for the last chapters of this book.

> ***Note**: From the conceptual point of view, the set of specific productivity curves have a correlation with what we could call specific injectivity curves. However, on account of simplicity, this work only focuses on the first ones.*

In the next chapter, real cases are analyzed in which the relative permeability concept leads to definitely erroneous conclusions when used to describe fluid **production**.

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