Chapter II

"… Scientists, it should be clear, never learn concepts, laws, and theories in the abstract and by themselves.... ...A new theory is always announced together with applications to some concrete range of natural phenomena... ...the process of learning a theory depends upon the study of applications, including practice problem-solving both with a pencil and paper and with instruments in the laboratory ..."

T. S. Kuhn. "The Structure of Scientific Revolutions" . .

THEORY AND APPLICATIONS

CHAPTER CONTENTS AND OBJECTIVES

In the previous chapter we developed a simple model of porous medium in order to get acquainted with concepts associated with relative permeability curves. Yet, as we shall repeatedly see throughout this book, such model is based on assumptions that require a profound critical analysis. This analysis will reveal a series of inconsistencies directly linked to the notion of relative permeability. To that end, displacements in naturally occurring porous media are analyzed in this chapter, emphasizing the relationship between the data generation and it's latter use.

In other words we shall review

- ✓ Theoretical and experimental procedures to measure and calculate relative permeabilities.
- ✓ The data set and equations the reservoir engineer needs to describe fluids production of a reservoir.

The analysis will show some shocking divergences between what is calculated and what is needed for an adequate reservoir characterization

Developments in this chapter are mainly numeric and conceptual. At first some experimental results are presented. They are later analyzed "intuitively", and this intuitive analysis is compared to routine methodologies used to obtain relative permeability curves.

Appendices at the end of this chapter contain more rigorous and detailed support. Theoretical developments are documented and results used in the main core of the chapter are explained in detail.

PROCEDURES, NUMBERS AND CONCEPTS

For starters we will take a look at a typical water-oil displacement at laboratory scale, devoted to the generation of the well known relative permeability curves.

In these displacements it is common practice to compute fluid productions as a function of time, while water injection rate or pressure are kept constant.

Initially, samples are conditioned to Swirr (usually beginning with a sample 100% saturated with water) through de injection of oil until no more water is expelled from the sample. At this point only mobile oil is present in the porous medium. Once reached Swirr condition, water injection proceeds at constant pressure (in the present example) until no oil is being produced.

The standard name for this procedure is "Non stationary displacement at constant pressure"

Experimental Data

Table II – 1 summarizes the sample petrophysic and fluid properties, as well as displacement conditions.

Table II - 1

General Data

Section	11.22	cm^2
Length	6.55	cm
Porosity	17.5	%
Absolute gas permeability	25.94	mD
Irreducible water saturation	32.2	% PV
Oil effective permeability @ Swirr	21.45	mD
Oil viscosity	18.5	cP
Water viscosity	1.02	cP
Pressure difference	38.4	psi
Pore volume	12.86	cm^3
Residual oil saturation		% PV
Effective water permeability @ Sor		mD
Original oil in place (OIIP)	8.72	cm^3

Values registered during the test are included in Table II - 2. Produced volumes are computed as a function of time during water injection at a constant pressure of 38.4 psi.

Table II – 2

Data from Displacement Test

	Time	Wi		Np	
	[sec]	[cm^3]	PV	[cm^3]	PV
1	-	-	0.00	-	0.000
2	30.0	0.16	0.01	0.16	0.012
3	60.0	0.33	0.03	0.33	0.025
4	120.0	0.73	0.06	0.73	0.057
5	242.3	1.64	0.13	1.64	0.127
6	340.0	2.46	0.19	2.46	0.191
7	400.0	3.00	0.23	2.61	0.203
8	550.0	4.42	0.34	2.89	0.224
9	731.7	6.32	0.49	3.12	0.243
10	1,451.1	15.16	1.18	3.67	0.285
11	2,458.8	30.16	2.35	4.06	0.315
12	3,597.0	49.78	3.87	4.34	0.338
13	5,229.4	81.58	6.34	4.59	0.357
14	6,835.7	115.99	9.02	4.75	0.369
15	14,953.0	323.83	25.18	5.11	0.397
16	30,512.0	811.46	63.09	5.36	0.416

Some additional commentaries are relevant:

- ✓ In spite of the fact that the organic phase is identified as “oil”, routinely a refined fluid with the “proper” viscosity is used for displacements instead.
- ✓ Absolute water permeability was obtained after the sample was saturated completely with water.
- ✓ Swirr was determined during oil injection. Once all mobile water had been displaced, effective permeability was measured (Ko[Swirr]). It is important to notice that Ko[Swirr] is not determined during water injection but in a previous stage.
- ✓ Viscosities of both phases were determined before all displacement tests. The choice of a highly viscous organic phase is justified later.
- ✓ Water injection is made at constant pressure. This pressure is chosen large enough so that fluid redistribution due to capillary forces is negligible. *(**Note**: in this case a constant pressure approach was chosen. A constant rate could have been chosen instead)*

- ✓ It is worth noting that neither the residual oil saturation (Sor) nor the effective permeability at that saturation (Kw[Sor]) are determined directly from the data of the water injection test but, as will be discussed latter, by extrapolation of experimental values.

The volume of water injected (**Wi**) is not measured in a direct way but rather it is calculated using total fluids and oil production rates, assuming the fluids are incompressible under the test conditions. Oil production (**Np**) is directly measured using a phases separator (See Fig. II-3).

Notice that, in this example, for the first 340 seconds only oil is produced. This is the time it takes for the water to reach the "production" face of the sample. The Break-Through time (BT), as it is usually called, depends on many factors, including sample heterogeneity, applied pressure, and mobility ratio.

Plots

Fig II – 1 is a plot of water injected volumes (**Wi**) and oil production volumes (**Np**) from Table II-2. The tendencies found in both curves could be considered as being typical of displacement test performed in laboratories.

> ***Note**: Notice that each series has a different vertical axis. Total volumes (the right vertical axis) extend through 1,000 cc, while oil produced volumes are shown in the left vertical axis, reaching volumes not beyond 6 cc.*

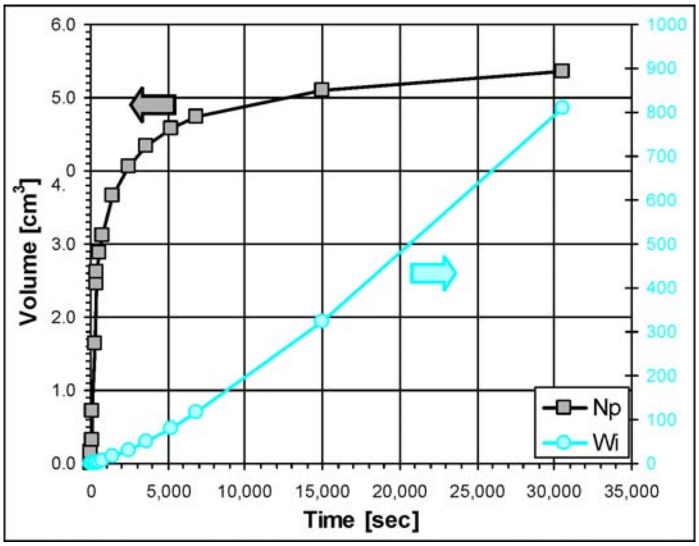

Fig. II-1: Water injection and oil production volumes during water injection test

Fig. II-1 displays volumes as a function of time. Yet we are interested in rates as a function of time or saturations. Rates are obtained by differentiating each volume vs. time curve, after an appropriate fit was found.

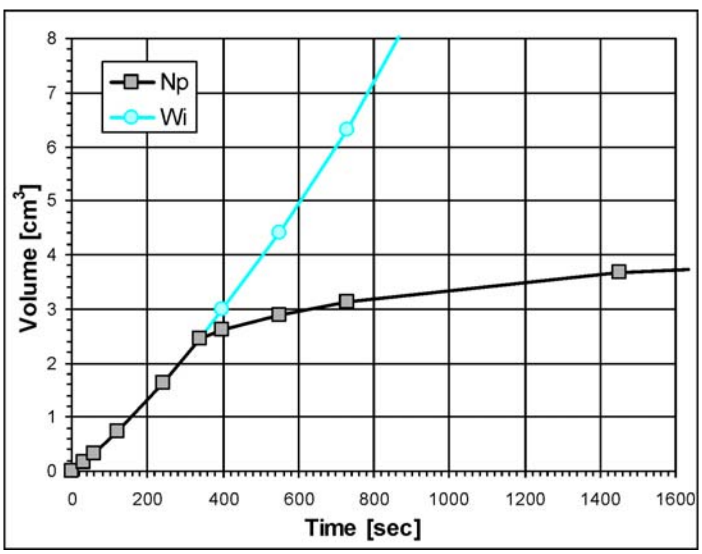

Fig. II-2: First 1,600 seconds of the water injection test.

Although the test lasted some 30,000 seconds (slightly more than 8 hours), Fig. II-2 shows a detail of the first 1,600 seconds. This allows us to watch oil production and water injection volumes in the same scale. Note that while the injection curve doesn't have significant change in it's tendency, oil production curve has a blatant slope change at the water break-though.

The main change of the production curve in the breakthrough is the change in curvature. It goes from concave up to concave down. This change is not gradual. Both theoretically and empirically oil production rates experience a discontinuity at the breakthrough.

Fitting Experimental Data

Table II – 3

Values Calculated Through Numerically Fitted Data

	T Sec	Wi cm^3	Np cm^3	Total Injection Rate cm^3/sec	Oil Production Rate cm^3/sec	Water Production Rate cm^3/sec
1	0	0.00	0.00	0.00475	0.00475	0
2	20	0.10	0.10	0.00532	0.00532	0
3	52	0.28	0.28	0.00597	0.00597	0
4	84	0.48	0.48	0.00645	0.00645	0
5	116	0.69	0.69	0.00684	0.00684	0
6	148	0.92	0.92	0.00719	0.00719	0
7	180	1.15	1.15	0.00749	0.00749	0
8	212	1.40	1.40	0.00778	0.00778	0
9	244	1.65	1.65	0.00804	0.00804	0
10	276	1.91	1.91	0.00828	0.00828	0
11	308	2.18	2.18	0.00852	0.00852	0
12	**340**	**2.46**	**2.46**	**0.00874**	**0.00874**	**0**
13	**340**	**2.46**	**2.46**	**0.00874**	**0.00286**	**0.00587**
14	532	4.25	2.86	0.00989	0.00160	0.00828
15	833	7.45	3.23	0.01131	0.00096	0.01035
16	1,304	13.20	3.59	0.01305	0.00060	0.01245
17	2,040	23.62	3.93	0.01512	0.00037	0.01476
18	3,194	42.52	4.25	0.01754	0.00022	0.01732
19	4,999	76.80	4.55	0.02029	0.00012	0.02017
20	7,824	138.71	4.81	0.02335	0.00007	0.02328
21	12,246	249.76	5.03	0.02666	0.00004	0.02663
22	19,167	447.14	5.21	0.03013	0.00002	0.03011
23	30,000	793.97	5.36	0.03363	0.00001	0.03362

Experimental data are necessarily discrete. Only some data points are collected during the displacement. Yet, as mentioned before, it is necessary to make some type of curve fitting in order to differentiate volume vs. time curves and obtain fluid rates at any time.

In the appendices of this chapter, a detail is presented of the curve fitting for water injection and oil production volumes (after BT) as a function of time. Using this fit, Table II-3 was constructed.

Some commentaries on the table:

- ✓ It was assumed that injection volumes are equal to total produced volumes and that injected water and produced oil are equal before BT.
- ✓ Water production rates are calculated by subtracting oil rate from total rate.
- ✓ The first row of data may seem anomalous because rate is non-zero even though no fluid has been produced yet. These rates where obtained from differentiation of the curve used to fit the data. In other words, initial rates are extrapolated at time = 0, using the fitted curves.
- ✓ Rows 12 and 13 show different rates for a particular moment of time (BT). These values are obtained by fitting the data before and after the BT separately.

A Direct Calculation

Table II-1 summarizes the geometrical properties of the sample, as well as viscosity and flow conditions (applied pressure difference). On the other hand, Table II-3 presents water and oil production rates. So, it would seem that with all the available data, we could determine effective permeability of each phase using Darcy's Equation:

$$\mathbf{Kw = Q_w \mu_w L / (A\, \Delta P)} \quad \text{Eq. II-1}$$

$$\mathbf{Ko = Q_o \mu_o L / (A\, \Delta P)} \quad \text{Eq. II-2}$$

Where

- ✓ **Kw** = Effective permeability to water.
- ✓ **Ko** = Effective permeability to oil.
- ✓ **Qw** = Water rate. (From Table II-3)
- ✓ **Qo** = Oil rate. (From Table II-3)
- ✓ **μw** = Water viscosity. (From Table II-1)
- ✓ **μo** = Oil viscosity. (From Table II-1)
- ✓ **ΔP** = Applied pressure difference. (From Table II-1)
- ✓ **A** = Sample Area. (From Table II-1)
- ✓ **L** = Sample Length. (From Table II-1)

Observations:

To write the previous equations the following assumptions and simplifications have been made:

- ✓ Considering the size of the sample and the flow geometry, it was assumed that gravitational forces are negligible when compared with the external displacement force.
- ✓ It was assumed that capillary phenomena are not significant. This is a common assumption and allows us to use the same pressure difference for both phases.
- ✓ Although equations "Eq. II-1" and "Eq. II-2" look like Darcy's equation for horizontal linear systems, actually they are not because $\mathbf{Q_w}$ y $\mathbf{Q_o}$ are production rates and do not represent the conduction capacity, but rather the capacity to produce fluids at the output face.

The magnitude of the last remark will be evident during the following developments. By now, considering the last observation, and to maintain a consistent notation, we will replace **Kw** and **Ko**, which are associated with **conduction**, by **Pw** and **Po** which are linked to the **production** capacity.

Hence, we can re-write the flow equations:

$$\mathbf{Pw = Q_w \mu_w L / (A\, \Delta P)} \quad \text{Eq. II-3}$$

$$\mathbf{Po = Q_o \mu_o L / (A\, \Delta P)} \quad \text{Eq. II-4}$$

where

- ✓ **Pw** = Effective water productivity.
- ✓ **Po** = Effective oil productivity.

A dimensional analysis can easily show that effective productivity has units of squared length, just as permeability. In fact permeability is a measurement of the **conduction** capacity of fluids and productivity evaluates the **production** capacity of the same fluids. Both magnitudes are related to fluids movement.

The similarity between both expressions goes further, since absolute permeability is equal to absolute productivity. This can be easily understood given the fact that when only one fluid circulates on the porous media, **conduction** and **production** rates must be equal due to conservation of mass, again assuming fluids are incompressible.

Just as it is done with permeabilities, we will define specific productivity to water (**Psw**) as the ratio between effective water productivity (**Pw**) and absolute productivity (**Pabs**). As mentioned earlier, the later coincides with the absolute permeability (**Kabs**)

Analogous definitions will apply to oil. Specific productivity to oil (**Pso**) will be the ratio between effective oil productivity (**Po**) and absolute productivity (**Pabs**).

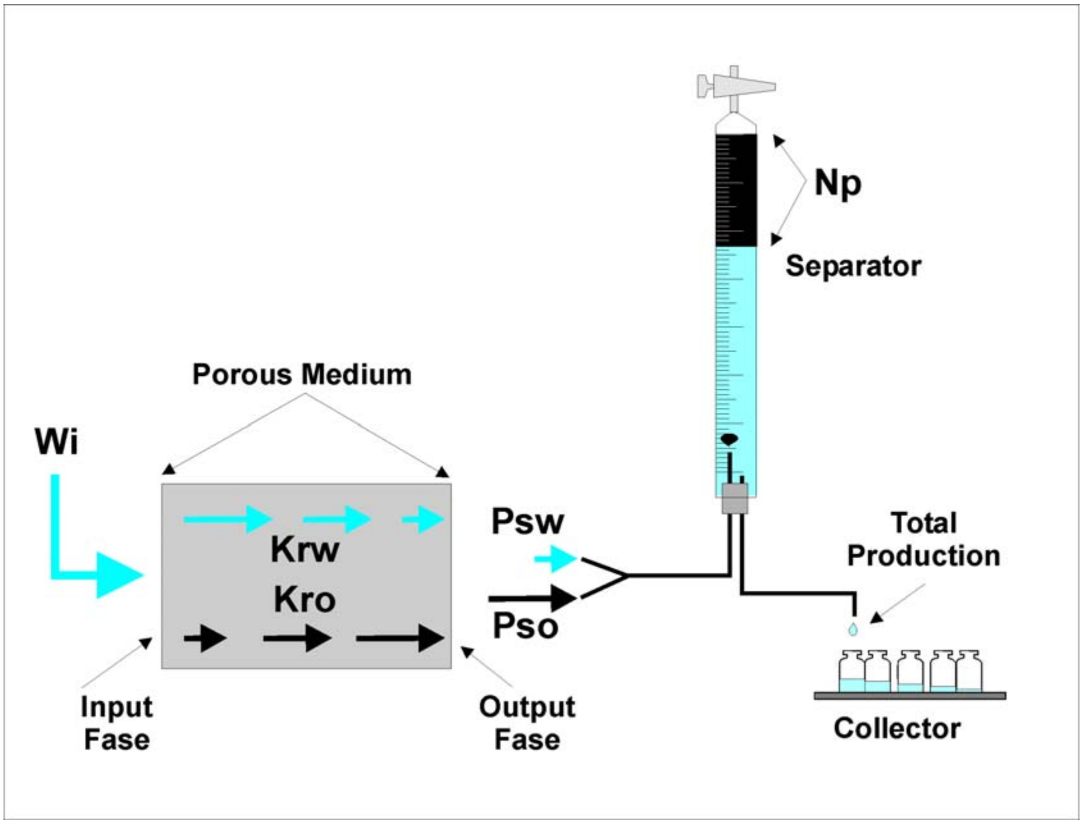

Fig. II-3: Schematic equipment used for water-oil displacement test.

Fig. II-3 shows a scheme of a typical equipment used during a water injection test. As it is emphasized in the figures, relative permeabilities apply within the porous media (fluids **conduction**) while specific productivity curves (**SPC**) are associated to production rates on the output face of the porous media (fluids **production**)

Results

Productivity equations (Eq. II-3 & Eq. II-4) can be used to calculate the values in Table II-4:

Table II – 4

Specific Productivity

	Avg. Sw [% PV]	Pso	Psw
1	32.20	0.757	0.000
2	32.99	0.848	0.000
3	34.39	0.951	0.000
4	35.94	1.027	0.000
5	37.60	1.090	0.000
6	39.34	1.145	0.000
7	41.17	1.194	0.000
8	43.07	1.239	0.000
9	45.04	1.281	0.000
10	47.07	1.320	0.000
11	49.16	1.357	0.000
12	51.30	1.392	0.000
13	51.30	0.457	0.052
14	54.46	0.256	0.073
15	57.33	0.153	0.091
16	60.08	0.095	0.109
17	62.75	0.059	0.130

18	65.28	0.035	0.152
19	67.58	0.020	0.177
20	69.60	0.011	0.205
21	71.30	0.006	0.234
22	72.71	0.003	0.265
23	73.85	0.002	0.295

For each point a material balance was made in order to calculate the average water saturation in the poral system:

$Swm_i = Swirr + 100\ Np_i / PV$ ....Eq. II-5

Where:

- ✓ Swmi = Average water saturation at point i.
- ✓ Swirr = Irreducible water saturation.
- ✓ Npi = Cumulative Oil production up to the point i.
- ✓ PV = Poral Volume.

Fig. II-4 is a plot of the data from Table II-4

As it could be expected, there is an abrupt change in the tendencies when BT occurs:

- ✓ Specific oil productivity increases all the way to the BT. This is expected because high viscosity oil (18 cP) is being replaced by water (1 cP), hence increasing average fluid mobility within the porous media. Since the pressure difference between input and output faces remains constant, higher production rates are obtained.
- ✓ After BT, specific productivity to oil drops drastically.
- ✓ Specific productivity to water remains zero until BT. This is a trivial result, in concordance with the definition of BT.
- ✓ Beyond BT specific water productivity increases monotonously as oil is driven out of the porous medium.

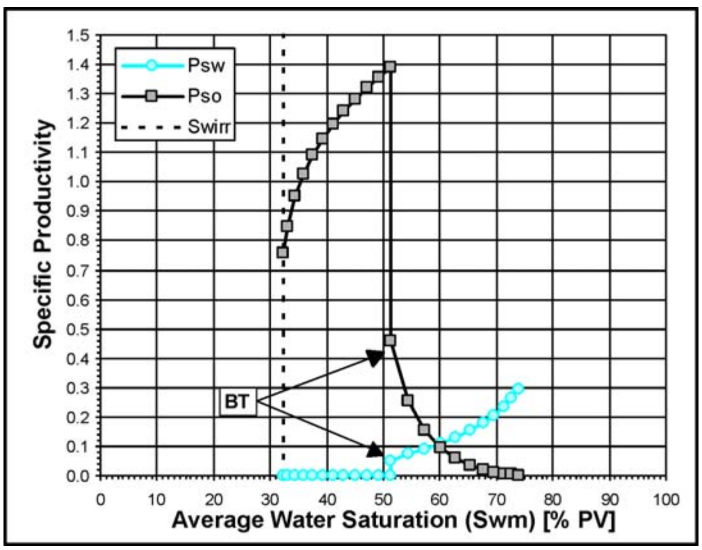

Fig. II-4 Specific Productivity Curves (SPC).

Summarizing it can be said that Fig. II-4 shows the capacity of the system to produce fluids as a function of average saturation during a water injection test. Yet, the trained eye will quickly notice that theses curves are very different from relative permeability curves. Evidence of this is the presence of values higher than “1” (one) and the abrupt change in tendencies.

In fact, with the data presented in Table II – 2 a specialized laboratory would have made a much more complex calculation, in order to end up reporting the relative permeability curves shown in Fig. II-5

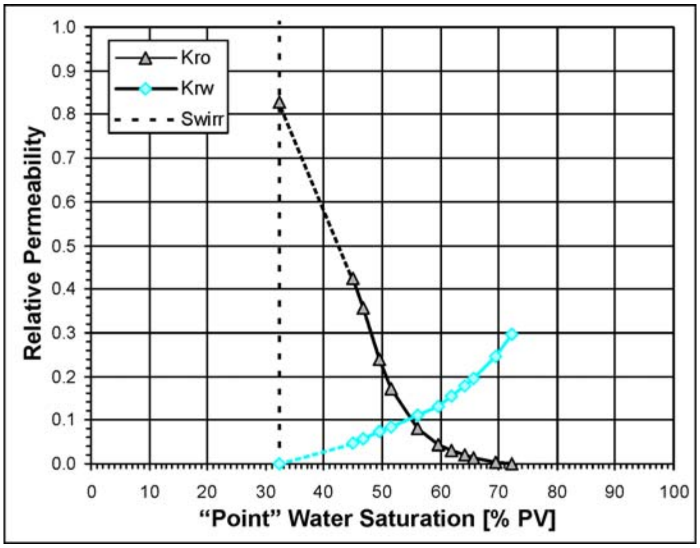

Fig. II-5: Relative permeability curves.

Fig. II-5 can be thought of as a typical set of relative permeability curves. They present the following features:

- ✓ No value of relative permeability is ever higher than 1.
- ✓ Curves show no sudden changes in their tendencies: They are monotonous.
- ✓ There is a Sw zone where relative permeability values are not reported. This zone is indicated with dotted line and corresponds to values of Sw between Swirr and what is know as the front saturation. This point, intimately related to BT, is discussed in the appendix of this chapter.
- ✓ Relative permeability values are reported as a function of "point" water saturation (that is the saturation on an infinitely thin slice of sample) and not average water saturations.

In spite of what has been stated, relative permeability and the here presented as specific productivity curves do have some analogies. Fig. II-6 shows both sets of curves. Similarities are apparent:

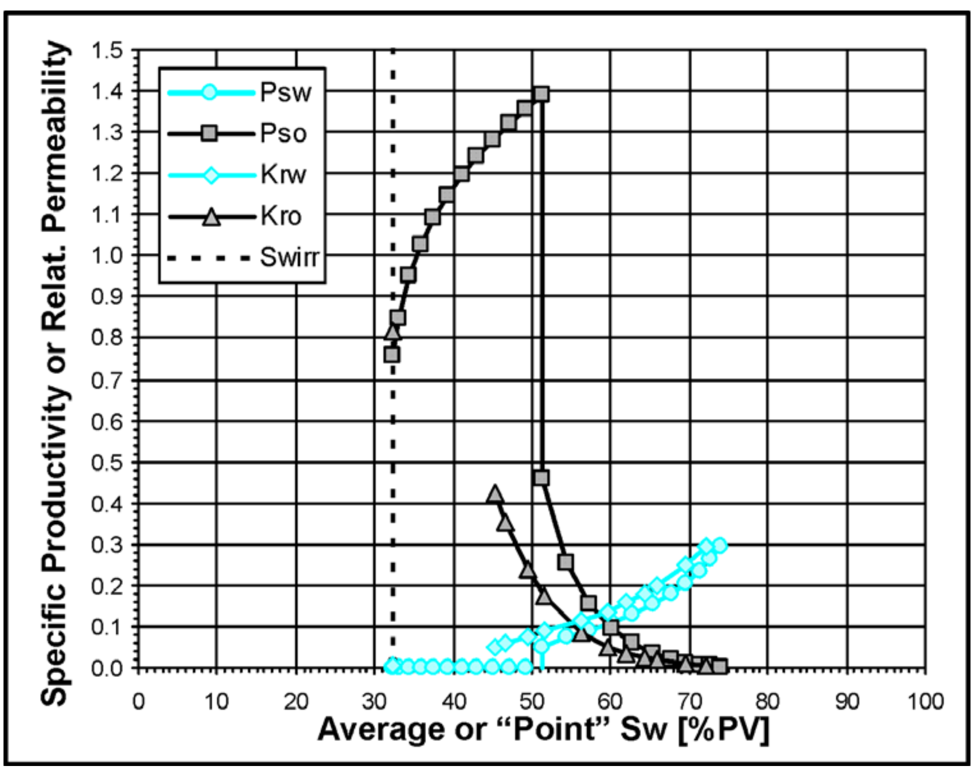

Fig. II-6: Comparison between relative permeability curves and SPC curves

Table II-5 indicates the differences and similarities between these sets of curves.

Table II – 5

Comparison between relative permeability and SPC

Property	Relative Permeability	SPC	Commentaries
Saturation end points	Experimental values of Swirr and Sor	Experimental values of Swirr and Sor	In both cases, end points correspond to the complete displacement of one phase by the other.
BT condition	Front water saturation	Mean water saturation	Mean water saturation is always higher than production face saturation. This is why BT happens at higher values of Sw in SPC than in KR curves.
Tendencies after BT	Monotonous	Monotonous	Tendencies are similar
Tendencies before BT	Curves not defined in this Sw zone	Zero for water and increasing or decreasing for oil	Although tendencies in **SPC** seem anomalous, they have direct physical interpretation.
Abscise	“Point” water saturation	Average water saturation	Reservoir calculations **always** use average values.
Type of Flow	Conduction	Production	Reservoir calculations are always aimed at prediction of fluids **production** Data available for history matching is always **production** history.

We are now in conditions to define more rigorously what relative permeability curves and **SPC** are:

- ✓ Relative permeability curves describe the capacity of a porous medium to conduct different phases within the porous medium as a function of point saturation.
- ✓ **SPC** describe the capacity to produce different phases as a function of average saturation of the porous system.

It seems that **SPC** are more suited to the needs of the reservoir engineer, since one of the main tasks he has to face, while modeling, is to describe fluid productions as a function of average water saturation for the reservoir itself or for every “block” used for discretization purposes.

- ✓ While doing material balance, production comes from the wells and average saturation is calculated from the material balance itself.
- ✓ During numerical simulation, production of each cell is estimated on each face and related to the average cell saturation.

Actually, if we where to ask ourselves what set of curves does the reservoir engineer need to describe the dynamic behavior, the answer is quite clear: **The reservoir engineer needs curves that express the capacity to produce different fluids as a function of average variables on the reservoir or the “block” under study.**

To the differences between curves just presented in Table II-5, an additional one must be added:

- ✓ Relative permeability curves only make physical sense when viscous forces are dominant. Although inherent to the very definition of relative permeability, this expression could seem outstanding. In the next chapter this limitation is analyzed in detail and posterior chapters show the errors that can be incurred when using relative permeability curves when other forces (capillary, gravitational) during displacement are not negligible.
- ✓ The **SPC** always have physical meaning, no matter what the acting forces are.

This difference is very important and we shall devote most of the following chapters to document it. In fact it is remarkable to notice that very few reservoir engineers are aware of this serious limitation behind relative permeability concept.

Numerical Simulation of the Displacement Test

In order to further illustrate the concepts just introduced, we shall analyze a simple case of numerical simulation with the data presented for the water displacement test. This simulation will allow us to compare the merits of **SPC** and relative permeability curves.

We will attempt through this way to show the limitations of the assumptions routinely made while using a reservoir simulator. More precisely, the following notions:

- ✓ Numerical dispersion is inherent to numerical simulation
- ✓ In order to improve the output of the simulator a larger number of cells is needed.

Should be changed for:

- ✓ Numerical dispersion is just a visible symptom of a poor description of the motion of fluids in porous media.
- ✓ In homogenous systems in which viscous forces are dominant the quality of the simulation does not depend on the number of cells

The following clarifications are relevant for the second statement:

- ✓ This statement is true only if relative permeability curves are replaced by **SPC** because using relative permeability curves the description of the flow process is never correct, whatever the number of cells used in the simulator.
- ✓ If capillary or gravitational forces are significant and affect fluid distribution, vertical refinement and not horizontal refinement should be emphasized
- ✓ In heterogeneous systems, the number of cells should not be lower than that required to make an appropriate description of the heterogeneity. In other words, the criterion should be geological and not mathematical.

To illustrate the previous statements, a simulation of the test using one cell was performed. As mentioned earlier, we intend to compare the descriptions made by the different sets of curves. The simulator used is very simple (generated on a spreadsheet) and its development is detailed in the appendix of this chapter.

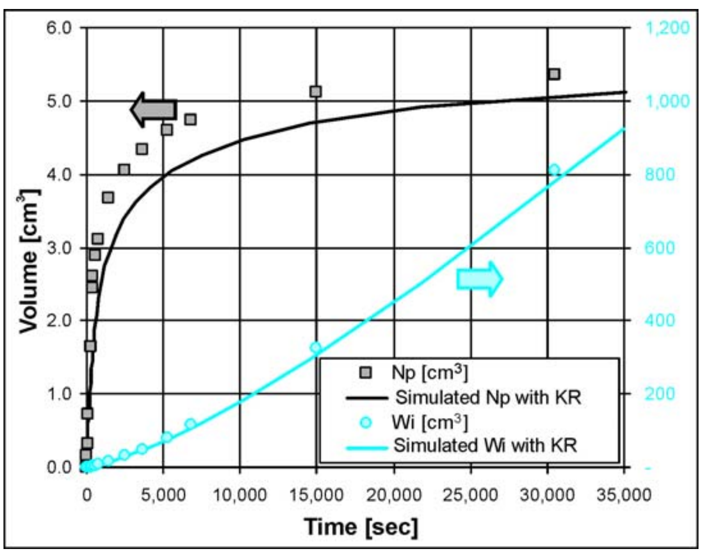

Fig. II-7: One cell simulation using relative permeability curves.

Fig. II-7 shows the result of the simulation using relative permeability curves. As it can be seen, prediction differ significantly form actual productions.

Fig. II-8, on the other hand, is the result of applying the same simulation procedure, but using **SPC** instead of relative permeability curves. In this case that prediction is optimal, despite having used just one cell

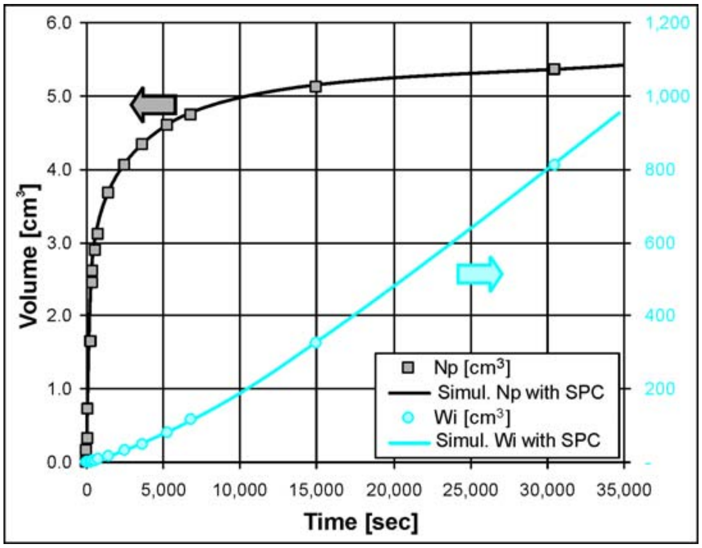

Fig. II-8: Numerical simulation using only one cell and SPC

Although the results are different enough, the difference in water cut predictions as a function of oil production is much more remarkable, as can be appreciated in Fig. II-9.

The predictions obtained using **SPC** matches the data perfectly whiles the relative permeability approach seems to give a really poor description of the process. The following comments are in order:

- ✓ The fast increment of water cut is due to numerical dispersion, as mentioned earlier.
- ✓ Inappropriate prediction, even in advanced stages of the simulation, is a consequence of trying to describe a production phenomenon using curves designed to describe conduction.
- ✓ The change in slope observed in the tendency of the fw curve calculated using relative permeability curves is caused by the linear interpolation used in the zones where relative permeability is not defined (between Swirr and SwBT). The use of other extrapolation techniques improves the aspect of the curve, but not its description of the physical phenomenon.

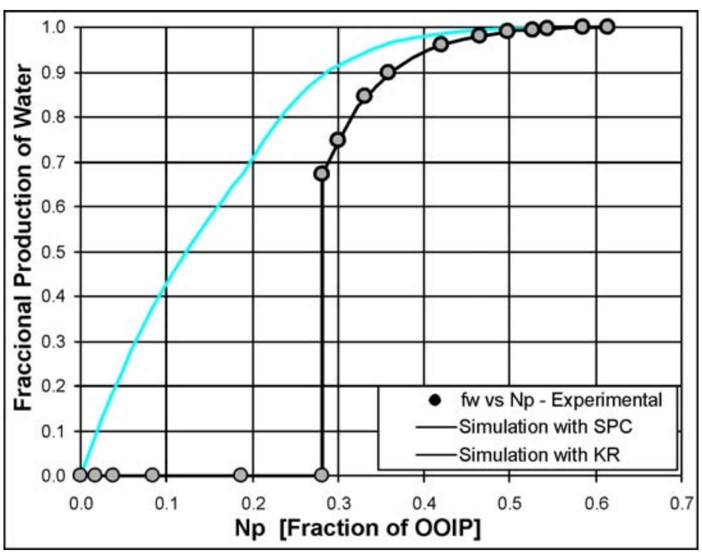

Fig. II-9: Comparison of the water cut predictions for simulators working with SP and relative permeability curves, using only one cell.

These type of results are amply discussed in a publication by Crotti and Cobeñas[1]. In this case, a numerical simulation was performed using commercial software with several numbers of cells, making improvements in relative permeability curves to minimize numerical dispersion.

Contrary to the common tenet amongst reservoir engineers, it is clearly illustrated that increasing the number of cells does not forces a convergence towards the "real behavior" of the reservoir, nor does it increase the "goodness" of its physical description.

On the other hand, **SPC** (not mentioned under this name in the quoted work) make an excellent description of the phenomenon, even with a linear model of just three cells!

FAQs

This section is a compendium of typical questions (and their answers) that have been asked during the regular presentations the author has made on the subject.

Commentary: The results obtained by the simulation where totally predictable since it is only a reversal of the process used to calculate the **SPC**. It's obvious that being the curves derived from the production data, reversing the process will reproduce exactly the initial data.

Answer: Of course! ... In fact, what must be putted under questioning is the regular use or relative permeability curves for production estimation. This example is just introduced to show the variables really needed for a successful numerical simulation. As just verified these variables are Average Cell Saturation and Production ability through output faces.

Question: The ideas presented in this chapter imply that some fundamental part of the reservoir engineering theory (and hence practice) needs to be changed?

Answer: No, the concepts don't need to be changed. All that has to be changed is the ways in which fluid movement is described. Present day equations where made to describe **conduction** of fluids while the reservoir engineer needs to evaluate **production** of these fluids.

Question: You repeatedly mention the difference between **conduction** and **production** but: Don't you **produce** only what you **conduct**?

Answer: No doubt, in order to produce a fluid you must conduct it through the porous medium inevitably. Yet the reciprocal of this statement isn't necessarily true: Through the life of a reservoir, important volumes of fluids are conducted and never produced. Besides, a long period of time may pass between **conduction** and **production** for a certain volume. A simple illustration of this is water injection. There may be a period of several months before the injected water emerges on the production well.

Question: Why is that laboratories report relative permeabilities in the full range of mobile saturation when, as stated in this chapter, they are not defined in the Swirr < Sw < SwBT range?

Answer: Labs usually report a dotted line in the undefined area of saturations, just indicating that there are no measured values in this range. This line does not imply that relative permeabilities in the Swirr < Sw < SwBT range have any physical meaning. It's just a line drawn on a paper!

Question: But…. Reservoir simulator use the complete relative permeability curve, including the Swirr < Sw < SwBT range!

Answer: True. But this is because simulators use average values within cells (despite using relative permeabilities). And average saturations between Swirr and SwBT **do** exist (point saturations do not!!).

Question: So, what curve should be feed to the simulators?

Answer: Numerical simulation is aimed at describing the relation between average saturation in a cell and it's capacity to product fluids on each face of the cell. Considering this fact, it seems logical to use **SPC**. The use of relative permeability curves, extrapolated to cover all values of Sw (extrapolated using the way you may wish), will always give an erroneous description of the process

Question: The **SPC** must be obtained from the lab?

Answer: Not only at labs. The curves so far presented describe the relationship between average saturation and productivity for the case of a linear geometry and one single cell where viscous forces dominate the displacement. The **SPC** for each cell depend on several factors, which are discussed in later chapters.

Question: But.... if **SPC** from the lab cannot be used directly in the simulator, what is the point of obtaining them?

Answer: Lab curves are the first approximation to solve the problem. This chapter shows that relative permeability curves NEVER lead to an adequate description of fluid production. The **SPC** to be used are to be constructed using laboratory data as well as reservoir information (production history, geology of the trap, etc)

Question: What is the use of relative permeability curves then?

Answer: Applications are scarce. They have conceptual value to understand fluid distribution within the porous medium, as shown in the appendix to this chapter.

Question: How can **SPC** be fed to a Simulator? They don't usually admit curves with discontinuities or values greater than 1!

Answer: Limitations in commercial simulators are aimed at avoiding errors while entering the data. Historically, it has been accepted that relative permeabilities (smooth, monotonous and smaller than 1) cover all the needs for description of fluid transport between cells. In this book we show, that real productions show abrupt changes and that, if you inject a more mobile fluid, total mobility is increased.

Question: What can be done until simulators begin to accept these "anomalous" curves?

Answer: In the meantime we must solve the compromise between limitations imposed by the software and the physical description needed.

Question: Labs that use the implicit method of calculation say that complete curves of relative permeabilities are obtained from non-stationary displacements. Does this solve the simulation problems?

Answer: This is not entirely true. The curve that is obtained is not complete. What is actually happening is that they define a family of complete curves and choose, from within this family, the curve that best fits the experimental data. These curves lead to numerical dispersion and in fact, violates the theory of frontal advance by stating implicitly that there are point saturations in the Swirr<Sw<SwBT range. In other words, this method provides a set of curves lacking physical meaning.

Question: Does all of this mean that, despite the high level of development reached by the oil industry, wrong concepts are being used to describe fluid movement, which, to make matters worse, could be considered as the heart of our industry?

Answer: Weird as it may sound, the answer is yes! But this idea had already been presented by L. Dake : "...*More than 50 years of intellectual bewilderment could have been avoided if the first authors to write on the matter had acknowledged the results indicating that a certain range on saturations where non existent during displacement...*" (The Practice of Reservoir Engineering). Nevertheless, I think that even though this statement is a step in the right direction it's only a partial solution as showed in the previous work of Crotti and Cobeñas[1], and fully documented through this book. Only understating the magnitude of the crisis, which is actually very deep and it is rooted in the use of **conduction** concepts to describe **production**, that the problem could be solved.

SUMMARY AND CONCLUSIONS

This chapter shows that relative permeability curves are inappropriate to describe production of fluids in a porous medium.

The concept of relative permeability is closely linked to the capacity to **conduct** fluids for each state of saturation. This saturation is determined on an infinitesimally thin section of porous media and is usually called "**point saturation**".

Reservoir engineering needs to describe the capacity to **produce** fluids as a function of **average saturation** of the block under study. This requires the use of **SPC**.

A numerical example is presented showing **SPC** working on a simple model.

SPC have direct physical interpretation and eliminate numerical dispersion in simulators. Additionally, while relative permeability curves can only be used to describe viscous-force-dominated displacements, **SPC** are adequate to describe fluid movements under capillary or gravitational dominion, even in heterogeneous media.

Chapters VI & VII further extend the concept of **SPC**. Also their determination under forces other than viscous is explained there.

REFERENCES

1. Crotti M.A., Cobeñas R.H., "Scaling Up of Laboratory relative permeability Curves. An Advantageous Approach Based on Realistic Average Water Saturations", SPE 69394
2. Buckley S. E. and Leverett M. C.: "Mechanism of Fluid Displacement in Sands", Trans., AIME (1942) 146, 107-16.
3. Welge H. J.: "A Simplified Method for Computing Oil Recovery by Gas or Water Drive", Trans., AIME (1952) 195, 179-86.
4. Johnson E. F., Bossler D. P., and Naumann V. O.: "Calculation of relative permeability from Displacement Experiments", Trans. AIME (1959) 216, 370-372.

5. Jones, S. C. and Roszelle W. O., "Graphical Techniques for Determining relative permeability from Displacement Experiments", J. Pet Tech. (May 1978), 807-817.
6. Crotti M. A.: "Método de Ajuste Numérico para la Obtención de curvas de Permeabilidad Relativa a partir de experiencias de desplazamiento a presión constante, Simposio de Producción de Hidrocarburos - IAP. (Bariloche, Argentina, 1988).

APPENDIX

Although it is highly recommend, it is not strictly necessary to read this appendix to understand the concepts presented in this book. They are meant to document the results presented in the core of this chapter. Reading of this section will be more beneficial for those who are familiar with Frontal Advance Theory, or for those who wish to inquire in routine relative permeability calculation.

Measurements and Calculation of Relative Permeability Curves

Theory

The most popular lab technique for determination of relative permeability curves is known as the non-stationary method. This technique uses the frontal advance theory[2] for displacements dominated by viscous forces in linear and homogenous systems.

Works by Welge[3] and Johnson et al[4] set the basis for the methodology. Non-stationary calculations are extremely sensitive to the goodness of the fitting curve for the experimental data. This is so because, in order to calculate relative permeability curves, first and second order derivatives are needed.

Posterior work by Jones and Roszelle[5] presents a graphical method of calculation based on graphical fittings. The equations developed by these authors were used in this book to calculate the values plotted in Fig. II-5.

The equations are here presented in a most compact way. The difficulties to solve this set of equations in an explicit way are easy to appreciate. First and second derivatives and squared first derivatives require a very good fitting algorithm and fairly good experimental data.

$$Sw = Swirr + 100 \times \frac{Np}{PV} - \frac{100 \times Wi \times \dfrac{dNp/dT}{dWi/dT}}{PV} \quad \text{......Eq. II - 6}$$

$$Kro = \frac{Lenght \times 14{,}700 \times \mu_o \times dNp/dT}{Area \times \Delta P \times Kabs \times \left(1 + \dfrac{Np \times d^2Wi/dT^2}{\left(dNp/dT\right)^2}\right)} \quad \text{......Eq. II - 7}$$

$$Krw = Kro \times \frac{\mu_w}{\mu_o} x \left(\frac{dWi/dT}{dNp/dT} - 1\right) \quad \text{......Eq. II - 8}$$

> ***Note**: It is worth noticing in Eq.. II-6 that while only oil is produced, it happens that Np = Wi and dNp/dT = dWi/dT. Hence Sw =Swirr! This is a mathematical way of showing why relative permeability curves are defined for Sw>SwBT only. In fact, the first value different from Swirr obtained is SwBT. In other words, point saturations between Swirr and SwBT are not defined in relative permeability curves calculation. This obeys to the existence of a "front" of saturations that forces an instantaneous jump from Swirr to SwBT as it goes through the porous medium.*

The Quality of Measurements

Considering the volumes of oil involved in the experience analyzed in this chapter allows for the understanding of the delicacy and precision with which displacement test must be made.

- ✓ Initial volume of oil (OOIP) = 8.72 cm^3.
- ✓ Volume of oil produce at BT = 2.46 cm^3.
- ✓ Total volume of oil produced = 5.36 cm^3.
- ✓ Volume of oil produced alter BT = 2.90 cm^3.

The fourth volume (the difference between 5.36 cm^3 and 2.46 cm^3) is the oil produced once both phases are present in the production face. All the calculation made to obtain relative permeability curves are made while producing this little volume. And, to appreciate the full need for accuracy during the displacement test we should consider that:

- ✓ The produced oil readings precision is no better than 0.05 cm^3.
- ✓ The dead volume in pipes is hardly ever below 0.2 cm^3. In some cases, the author has seen dead volumes as large as 0.5 cm^3.
- ✓ The readings are dynamic (non-stationary system) and oil "travels" upward inside the separator (see Fig. II-3) as discrete drops of a volume of approximately 0.1 cm^3. This is relevant if you consider that at typical rates, reading can be made without computing the drops in transit.

What Happens During the Water Displacement Test?

This section is destined to those readers who wish to visualize in some detail what happens in a porous medium during displacement. The plots in this section were made using the calculation procedures presented in references 2, 3 and 5.

The Frontal Advance Theory predicts that a "saturation front" of the displacing fluid will form, and the value of this "jump" remains constant as the displacement progresses.

Fig II-10 depicts the displacement process before BT. It models the saturation profile as a function of distance after 120 seconds have passed.

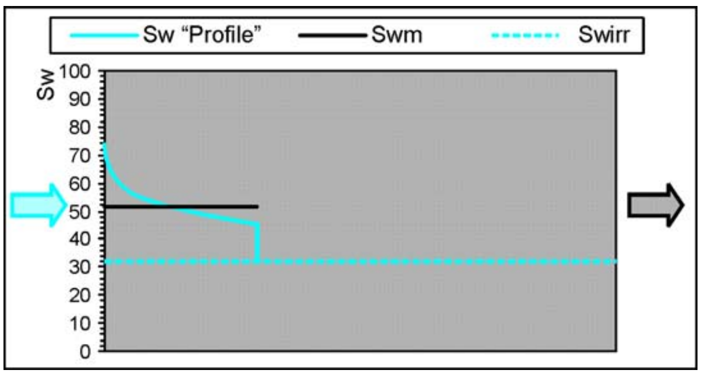

Fig. II-10: Saturation profile as a function of space after 120 sec. have passed

The abscise represents the length of the sample and the arrows indicate the direction of flow, blue meaning water injection and black meaning oil production in the end face. The ordinate shows water saturation.

The blue line is the saturation profile, according to the frontal advance theory, for the tested sample.

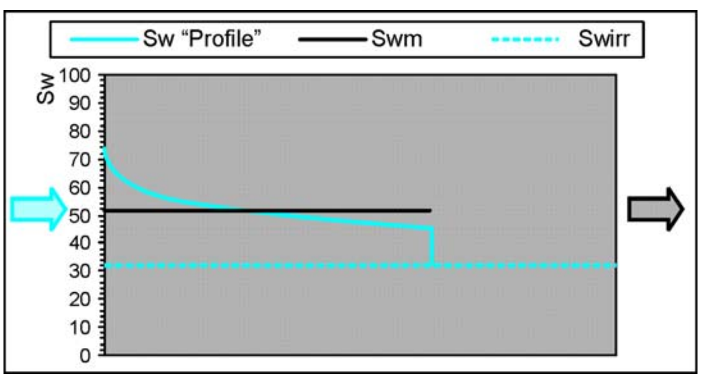

Fig. II-11: Saturation profile as a function of lenght after 242 sec. have passed

Fig II-11 shows a more advanced situation. In this case 242 sec. after test beginning.

In both figures, the horizontal black line represents the value of water average saturation behind the saturation front. According to the frontal advance theory, this value remains constant for all times before BT. Hence the saturation profile in this period simply stretches horizontally, maintaining its average value

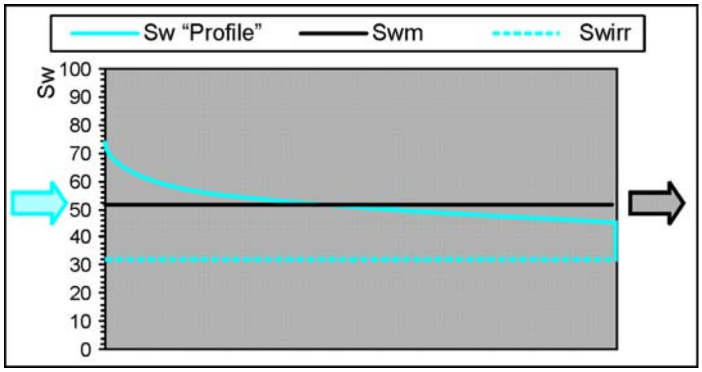

Fig. II-12: Saturation at BT. 340 sec. have passed.

As injection proceeds, BT is reached in 340 sec. (Fig. II-12). This time is the last point of monophasic production. Saturation in the end phase is close to 45% while average saturation is in the order of 51%. The volume injected to far is approximately 2.46 cm^3, equivalent to some 0.20 PV.

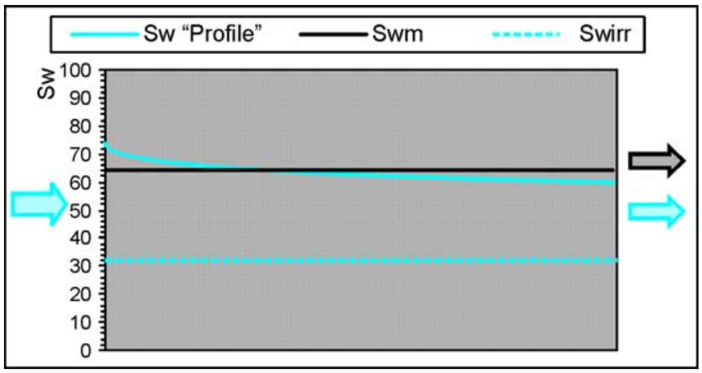

Fig. II-13: Profile after 2,459 sec. Both fluids are being produced in the end face

Fig. II-13 shows a much more advanced state of displacement. Injected volumes of water are 30.16 cm^3 (that is 2.35 PV). As expected, the saturation profile looks more and more like the average saturation. In fact, they will be equal once enough time elapse. Once this final state is reached, oil saturation is called residual oil saturation (Sor) and it occurs usually after several PV have been injected.

Water and Oil Production Rates

Fig. II-14 shows a numerical fit of experimental production data during the displacement test. Additionally, Fig. II-15 shows a detail of the first 1,600 sec. of the test (approximately 5% of the total experimental time) so that the quality of the fit can be appreciated.

The fit was made using a technique presented by Crotti[6] in 1988. The advantage of this type of fit is that high order polynomial can be used without having the oscillations polynomials usually present. This way, accurate values of first and second order derivatives can be calculated. Other types of data fit can be used but the one used here is extremely simple and accurate.

The general equation of the curve is: $Y = c_n X^n + c_{n-1} X^{n-1} + ... + c_1 X + c_0$

Where:

- ✓ Y = Log (Total produced volume)
- ✓ X = $(\text{Time} + a)^b$
- ✓ $c_n ... c_0$ = Polynomial coefficients
- ✓ n = Degree if polynomial
- ✓ a = Displacement parameter.
- ✓ b = Shape parameter.

The fitting procedure is as follows:

1. Find "a" and "b" such that the linear correlation coefficient is maximized.

2. Select a polynomial degree and find the polynomial coefficients using least squares methods. Change the polynomial degree until you find the one that best describes the values and tendencies

In this case the output was:

- ✓ n =5
- ✓ a = 1.2671E+02
- ✓ b = -1.7807E-01
- ✓ c_0 = 7.7235E+00
- ✓ c_1 = -4.5326E+01
- ✓ c_2 = 1.2766E+02
- ✓ c_3 = -2.4855E+02
- ✓ c_4 = 2.6888E+02
- ✓ c_5 = -8.7946E+01

***Note**: These values are only valid for the units used in the example (time in seconds and volumes in cm^3)*

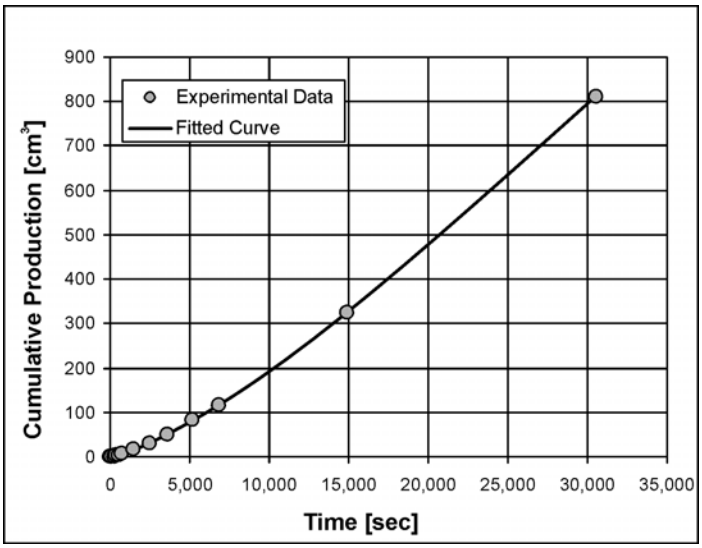

Fig. II-14: Data fit of total produced volumes as a function of time.

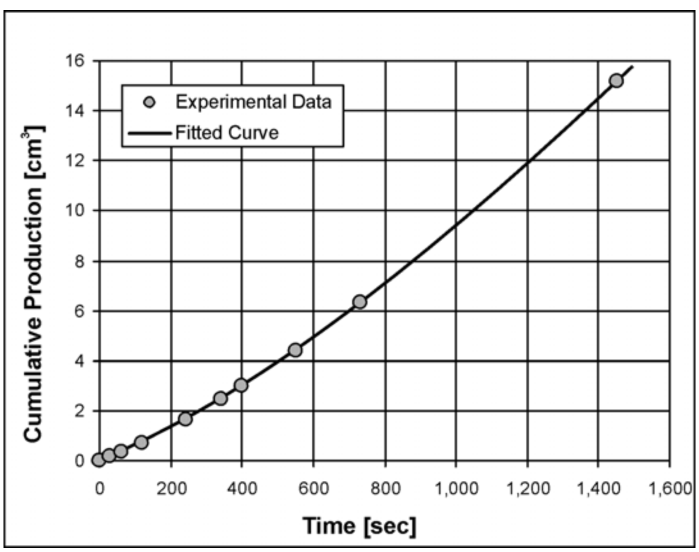

Fig. II-15: Detailed view of data fitting of total volumes as a function of time at the beginning of the displacement.

Table II-6 compares total fluid production experimental values with predicted values.

Table II-6

Numerical Fitting of Total Fluids Production

	Time	Total Production $[cm^3]$		Difference	
	[sec.]	Experim.	Fitted	$[cm^3]$	[%]
1	-	0.00	0.00	0.00	-
2	30.0	0.16	0.16	0.00	0.39
3	60.0	0.33	0.33	-0.01	-1.69
4	120.0	0.73	0.72	0.01	0.91
5	242.3	1.64	1.64	0.00	0.04
6	340.0	2.46	2.46	0.00	0.00
7	400.0	3.00	2.99	0.01	0.33
8	550.0	4.42	4.43	-0.01	-0.18
9	731.7	6.32	6.32	-0.01	-0.10
10	1,451.1	15.16	15.16	0.00	-0.01
11	2,458.8	30.16	30.15	0.01	0.03
12	3,597.0	49.78	49.74	0.04	0.08
13	5,229.4	81.58	81.52	0.06	0.07
14	6,835.7	115.99	116.09	-0.11	-0.09
15	14,953.0	323.83	324.08	-0.25	-0.08
16	30,512.0	811.46	811.22	0.24	0.03

It can be appreciated that up to point #11, the fitted curve reproduces experimental data with an error smaller than 0.01 cm^3. For later times, errors are not bigger than 0.1% of the total produced volume. Hence the description made by the fitted curve can be considered satisfactory in the full range of experimental values.

As for total production volumes, Fig. II-16 shows a fitted curve for the oil production data, starting at BT. Oil production before BT is directly obtained from the total production curve, which was fitted in the previous section.

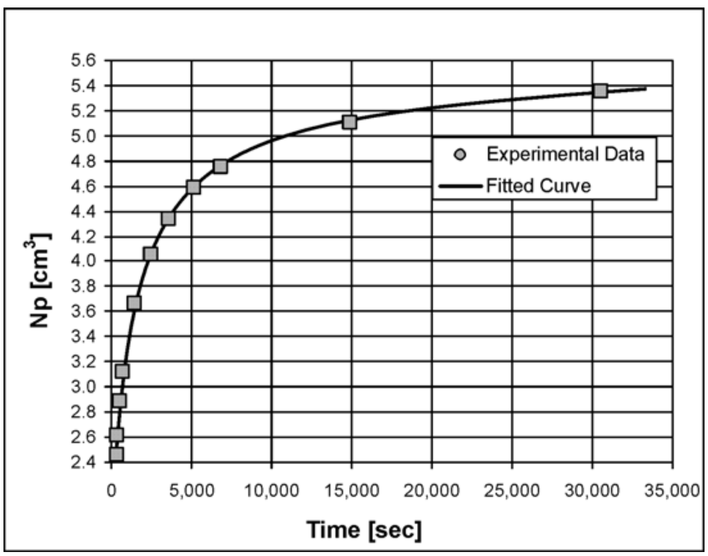

Fig. II-16: Curve fit of oil production as a function of time

As was made in the previous case, Fig. II-17 shows a detail of the first 1,600 sec. of the test so that the quality of this new fit can be also appreciated.

Once again, the numerical Fit was made using the equation: $Y = c_n X^n + c_{n-1} X^{n-1} + ... + c_1 X + c_0$

Where, in this case:

- ✓ Y = Oil produced volumes
- ✓ X = $(\text{Time} + a)^b$

- ✓ $c_n \ldots c_0$ = Polynomial coefficients
- ✓ n = degree if polynomial
- ✓ a = displacement parameter.
- ✓ b = shape parameter.

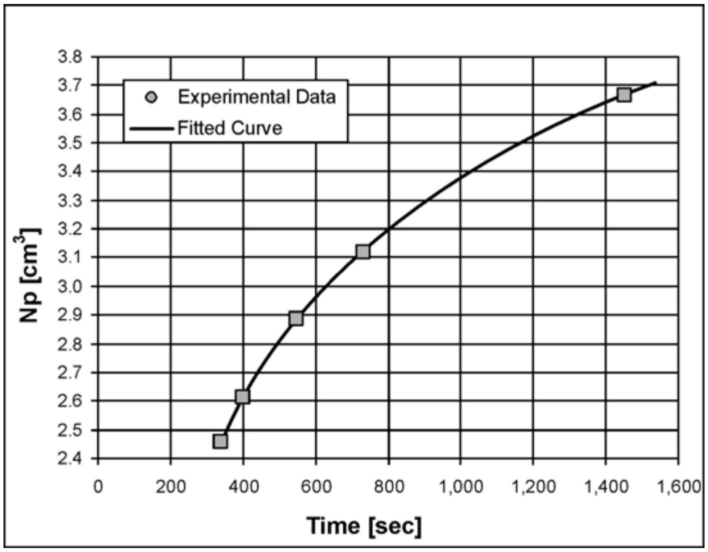

Fig. II-17. Detail of the first 1,600 sec. of the curve fitting of oil production as a function of time

The best values, after the optimization of "a" and "b", are:

- ✓ n = 3
- ✓ a = 2.0000E+02
- ✓ b = -6.0000E-01
- ✓ c_0 = 5.8837E+00
- ✓ c_1 = -2.7561E+02
- ✓ c_2 = 9.3927E+03
- ✓ c_3 = -1.6959E+05

Table II-7 presents a comparison between measured and fitted values. All errors are within the expectable range due to measurement uncertainties (± 0.01 cm^3). Once again the obtained fitting can be considered as being highly satisfactory.

Table II-7

Numerical Fit of Oil Production

	Time	**Oil production [cm^3]**		**Difference**	
	[sec.]	**Experim.**	**Fitted**	**[cm^3]**	**[%]**
6	340.0	2.46	2.46	0.00	0.00
7	400.0	2.61	2.61	0.00	0.08
8	550.0	2.89	2.89	-0.01	-0.19
9	731.7	3.12	3.13	-0.01	-0.25
10	1451.1	3.67	3.67	0.00	-0.09
11	2458.8	4.06	4.07	-0.01	-0.30
12	3597.0	4.34	4.34	0.01	0.12
13	5229.4	4.59	4.58	0.01	0.23
14	6835.7	4.75	4.74	0.01	0.20
15	14953.0	5.11	5.11	0.00	-0.10
16	30512.0	5.36	5.36	-0.01	-0.10

Numerical Simulation Using a Spreadsheet

This section is meant to explain how the values in figs. II-6 through II-8 where obtained.

The calculation sequence is the following:

1. Chose the Work columns.

   1.1. **Column B**: Sw at the beginning of the time step. In the first row the value of Swirr must be placed. For the next rows a constant increment of Sw was used in order to have 17 time steps until BT and 17 more from BT to the end

   1.2. **Column C**: Pressure difference between end faces. Although this value is constant throughout this particular test, the sheet doesn't require this condition.

   1.3. **Column D**: Time step. The first value is defined as zero. The following values are calculated based on the oil production required by column K.

   1.4. **Column E**: Relative permeability or SP value for oil for the corresponding value of Sw from column B. Values are obtained by linear interpolation.

   1.5. **Column F**: Relative permeability or SP value for water for the corresponding value of Sw from column B. Values are obtained by linear interpolation.

   1.6. **Column G**: Oil rate calculated using Eq. II-2 or Eq. II-4, depending on whether relative permeability or SPC are being used.

   1.7. **Column H**: Water rate calculated using Eq. II-1 or Eq. II-3, depending on whether relative permeability or SPC are being used.

   1.8. **Column I**: Total rate. It's the sum of columns G and H.

   1.9. **Column J**: Total injected (and produced) volume for the corresponding time-step. It's the product of the total rate times the time step. In the first row since □t = 0 then Injected volume is zero. In the following rows, rate should be obtained as an average between initial and final rates.

   1.10. **Column K**: Oil volume produced in the time step. This is calculated using material balance so that Sw has the value indicated in column B.

   1.11. **Column L**: Cumulative time.

   1.12. **Column M**: Cumulative Total volume.

   1.13. **Column N**: Cumulative Oil volume.

   1.14. **Column O**: Oil produced as a fraction of the OOIP. Divide column N by the OOIP.

   1.15. **Column P**: Water fractional flow. Subtract column G from column I and divide the result by column I.

2. Repeat the formulas in the next rows, considering the following:

   2.1. When BT is reached, use a small Sw increment to obtain results immediately after water production starts.

   2.2. The number of intervals of Sw was chosen arbitrarily. You can add or take intervals to vary overall precision.

The simulation can be carried out using other methods (fixed time step, for instance). The results would be very similar and have no influence on the conclusions we reached.

Tables II-8 and II-9 show the values obtained with this simple Simulator using **SPC**.

Table II-8

Numerical Simulation with a Spreadsheet - Columns B to I

	B	C	D	E	F	G	H	I
9	Sw [%PV]	ΔP [psi]	ΔT [secs]	Pso	Psw	Qo	Qw	Q total
10	32.200	38.4	0.00	0.757	0.000	0.004749	0.000000	0.004749
11	33.394	38.4	29.94	0.878	0.000	0.005508	0.000000	0.005508
12	34.588	38.4	26.63	0.960	0.000	0.006026	0.000000	0.006026
13	35.782	38.4	24.72	1.020	0.000	0.006398	0.000000	0.006398
14	36.976	38.4	23.46	1.067	0.000	0.006695	0.000000	0.006695
15	38.170	38.4	22.50	1.108	0.000	0.006956	0.000000	0.006956
16	39.364	38.4	21.71	1.146	0.000	0.007190	0.000000	0.007190
17	40.558	38.4	21.06	1.178	0.000	0.007391	0.000000	0.007391
18	41.752	38.4	20.51	1.208	0.000	0.007581	0.000000	0.007581
19	42.946	38.4	20.02	1.236	0.000	0.007758	0.000000	0.007758
20	44.140	38.4	19.59	1.262	0.000	0.007919	0.000000	0.007919
21	45.334	38.4	19.20	1.287	0.000	0.008074	0.000000	0.008074
22	46.528	38.4	18.85	1.310	0.000	0.008218	0.000000	0.008218
23	47.722	38.4	18.53	1.332	0.000	0.008356	0.000000	0.008356
24	48.916	38.4	18.23	1.353	0.000	0.008489	0.000000	0.008489
25	50.110	38.4	17.96	1.373	0.000	0.008614	0.000000	0.008614
26	51.3039	38.4	17.70	1.392	0.000	0.008737	0.000000	0.008737
27	51.3040	38.4	0.002	0.456	0.052	0.002865	0.005872	0.008737
28	52.713	38.4	70.14	0.367	0.061	0.002303	0.006947	0.009250
29	54.122	38.4	89.64	0.277	0.070	0.001741	0.008022	0.009763
30	55.531	38.4	116.71	0.218	0.080	0.001365	0.009054	0.010419

Table II-9

Numerical Simulation with a Spreadsheet- Columns J to P

	J	K	L	M	N	O	P
9	ΔVol Total [cm^3]	ΔVol Oil [cm^3]	Time [sec.]	Cumulative Total Vol. [cm^3]	Cumulative Oil Vol. [cm^3]	Np [OOIP]	Water fractional Flow
10	0.000	0.000	0.00	0.000	0.000	0.00000	0.00000
11	0.154	0.154	29.94	0.154	0.154	0.01761	0.00000
12	0.154	0.154	56.57	0.307	0.307	0.03522	0.00000
13	0.154	0.154	81.29	0.461	0.461	0.05283	0.00000
14	0.154	0.154	104.75	0.614	0.614	0.07044	0.00000
15	0.154	0.154	127.25	0.768	0.768	0.08805	0.00000
16	0.154	0.154	148.96	0.921	0.921	0.10566	0.00000
17	0.154	0.154	170.02	1.075	1.075	0.12327	0.00000
18	0.154	0.154	190.54	1.229	1.229	0.14088	0.00000
19	0.154	0.154	210.56	1.382	1.382	0.15849	0.00000
20	0.154	0.154	230.15	1.536	1.536	0.17611	0.00000
21	0.154	0.154	249.36	1.689	1.689	0.19372	0.00000
22	0.154	0.154	268.21	1.843	1.843	0.21133	0.00000
23	0.154	0.154	286.74	1.996	1.996	0.22894	0.00000
24	0.154	0.154	304.97	2.150	2.150	0.24655	0.00000
25	0.154	0.154	322.93	2.303	2.303	0.26416	0.00000
26	0.154	0.154	340.629	2.457	2.457	0.28177	0.00000
27	0.000	0.000	340.631	2.457	2.457	0.28177	0.67212
28	0.631	0.181	410.77	3.088	2.638	0.30255	0.75107
29	0.852	0.181	500.41	3.940	2.819	0.32333	0.82172
30	1.178	0.181	617.12	5.118	3.001	0.34411	0.86898