Chapter I

"Full rock relative permeabilities always seem to have been treated with great veneration throughout the history of reservoir engineering. They are assumed to be intrinsically correct and all theory and practice is geared to accommodate this commonly held view. In fact, as argued throughout the chapter, this is a questionable attitude and full rock curves are never used directly unless the problem in hand is that of flooding a reservoir which has the dimensions of a core plug and is full of 17 cp oil, which is a condition seldom encountered in practice."

L. Dake. " The Practice of Reservoir Engineering" p. 361

THE RELATIVE PERMEABILITY CONCEPT

The opening quotation could be considered sarcastic. However, conceptual and numerical analysis presented later in this book, will show that there is a high degree of plain truth in Dake's words.

To begin in an ordered fashion, we will first introduce the concept of relative permeability. We will use a simple and easy-to-understand model, omitting complex theories (that is, using only well established physical laws). Furthermore, this model will show the dependence that relative permeability curves have on gravitational effects and capillary phenomena.

PERMEABILITY OF POROUS MEDIA

Darcy's Law, named after French engineer Henry Darcy, describes the relationship between pressure head loss and flow rate in a homogenous porous media saturated with a monophasic fluid, which is moving through it.

In absence of gravity and for a linear geometry, fluid flow rates depend on:

- ✓ The geometry of the system: Area (**A**) and Length (**L**).
- ✓ Fluid viscosity (μ).
- ✓ Pressure head loss across the media (ΔP)

Experiments have shown that, other variables remaining constant, rate (**Q**) is proportional to **A** and Δ**P** and inversely proportional to μ and **L**. Thus:

$$\mathbf{Q = K\ A\ \Delta P / (\mu\ L)} \quad \text{............ Eq. I-1}$$

Where the proportionality constant **K** is known as permeability and must be considered a property of the porous media. In other words, any change in one or more variables, in the right side of the equation, leads to changes in flow rate in order to maintain the numeric value of **K**.

> ***Note**: Permeability can change as a consequence of clay swelling, fines migration or changes in net over burden pressure (NOBP). When this occurs, it is not accounted as a different **K** for the same porous medium but a change of porous medium, because its pore structure has been altered.*

Once firmly established that permeability is a property of the porous material (not influenced by the fluid, system geometry or flow conditions), we can write the following definition:

Permeability is the capacity of a porous medium to conduct fluids

In regular lab practice, **conduction** capacity is determined through direct measurement of either incoming or outcoming rates. That is, **conduction** is evaluated through **injection** or **production** capacities.

SIMPLIFIED MODEL OF POROUS SYSTEM

Naturally occurring porous media have complex, tortuous and interconnected capillary conducts which hinder us from finding simple functional relations between fluid distribution and flow rates. To avoid this problem, and help to visualize the flow characteristics, we will use a very simple capillary network.

The simplified capillary model consists of a square-based prism with longitudinal capillary conducts. The conducts are parallel to the longitudinal axis and have no interconnection, the flow capacity of such arrangement is independent of the presence or the state of the neighboring conducts.

The figures presented show only a section of the model, perpendicular to the longitudinal axis.

In order to develop numerical calculations, leading to relative permeability values, we should bear in mind the following points:

- ✓ The Area (**A**), in Darcy´s equation, is the bulk area, not the sum of the sections of the conducts
- ✓ Assuming a constant length, the volume in the capillary conducts is proportional to the second power of the radius.

  Volume of capillary tube $= \pi r^2 L$ .... Eq. I-2

- ✓ According to Poiseuille´s law, the capacity of a capillary tube to conduct fluids, in laminar regime, depends on the fourth power of the radius.

  $Q = \pi r^4 \Delta P / (8 \mu L)$ .... Eq. I-3

If only one tube is drilled in the prism, this only tube accounts for the porosity and permeability of the prism. The pore volume (PV) coincides with the volume of the orifice and the flow through it is the only flow through the whole system.

Should a second orifice be drilled, equal to the first, both fluid hold-up and fluid transport capacity are duplicated.

On the other hand, if only one orifice is drilled with twice the radius of the first, the pore volume increases 4-fold while the capacity to conduct fluids increases 16-fold!. This quantities are obtained when replacing $\mathbf{r}^2$ by $(\mathbf{2r})^2$ in Eq. I-2 and $\mathbf{r}^4$ by $(\mathbf{2r})^4$ in Eq. I-3.

With these primary concepts in mind it is possible to determine the properties of the porous system when new capillaries are added. We will arbitrarily use capillary tubes with radii 1, 2 and 3.

> ***Note****: Only the proportion between radii and not their absolute values are relevant to our analysis.*

Fig. I-1 shows the properties of a block with a ~~unit~~ radius equal one. The gray square represents the bulk area while the capillary tube gives the block its porosity and capacity to conduct fluids. Since the block has arbitrary dimensions, we will choose the first case to have porosity 0.1% and permeability 0.1 mD.

Fig. I-1: Block with just one orifice of radius 1.
Porosity = 0.1%
Permeability = 0.1 mD

> ***Note****: Values of permeability and porosity are arbitrary. Dimensions can be selected so that a block with these properties could be built.*

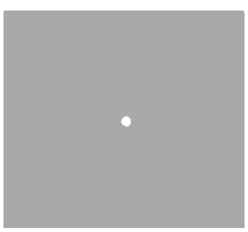

Fig. I-2: Block with one orifice of radius 2.
Porosity = $(0.1\% \times 2^2) = 0.4\%$
Permeability = $(0.1 \text{ mD} \times 2^4) = 1.6$ mD

> ***Note:*** *Permeability has increased much more than porosity*

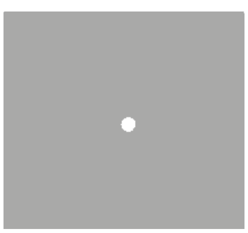

Fig. I-3: Block with one orifice of radius 3.
Porosity = (0.1% x 32) = 0.9%
Permeability = (0.1 mD x 34) = 8.1 mD

> ***Note****: Again, as the radius increases, porosity increases slowly compared with the increase in permeability,*

Models with only one orifice are oversimplifications and do not allow for an adequate simulation of naturally occurring porous media. In order to make our model more representative of a real porous medium, we will include several orifices of different radii within the same block. An example of this is shown in Fig I – 4 where there are 100 orifices of radius 1, 20 of radius 2 and 10 of radius 3.

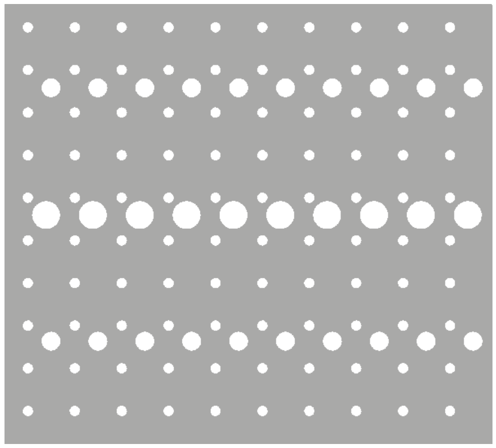

Fig. I-4: 100 orifices of radius 1, 20 of radius 2 and 10 of radius 3.

Considering that conductive and volumetric properties are additive (the tubes are not interconnected) a simple calculation will allow us to calculate the properties of this block:

Table I-1
Properties of the Model

Orifice	Quantity	Porosity	Permeability
Radius 1	100	10 % (100*0.1%)	10 mD (100*0.1 mD)
Radius 2	20	8 % (20*0.4%)	32 mD (20*1.6 mD)
Radius 3	10	9 % (10*0.9%)	81 mD (10*8.1 mD)
Total	130	27 % (10+8+9)	123 mD (10+32+81)

This simple model has several characteristics common with naturally occurring porous media:

- ✓ It has a wide distribution of pore diameters. Almost equal volumes are attributed to small, medium and large capillaries.
- ✓ Even though the pore volumes of the three pore families are similar (10%, 8% & 9%),, permeability is dominated by the wider channels. Pore channels of radius 3 account for only one third of the porosity, but represent almost 2/3 of the overall permeability.

RELATIVE PERMEABILITY

Biphasic Flow

In the next stage we will apply the model to analyze multiphase flow.

Let us start with our porous media saturated 100% with oil. As it was mentioned before, under these conditions, permeability to oil is 123 mD.

If we displace the oil with water, the system will conduct fluid according to the properties of the individual capillary tubes. To simplify, we will assume that there are no residual oil or irreducible water saturations (Swirr = Sor = 0).

Case I: Low rate displacement in water wet system

From a practical point of view, to say that a rock is water wet means that along the displacement, water will enter the smaller capillary tubes first. As the water saturation increases, the tubes are filled sequentially from the narrower to the wider ones.

> ***Note**: Through the following analysis every capillary "family" is filled in a gradual fashion. In fact, because each orifice is identical to any other orifice of the same family ("small", "medium" or "large") there are no reasons to make differences among them. The gradual filling is made attending to didactical purposes only. In this way, with only three families we can show what is expected in more complex systems with larger number of capillary radii. The main reason to use only three pore radii is the simplification of related calculations.*

The following figures show the consequences of increasing water saturation.

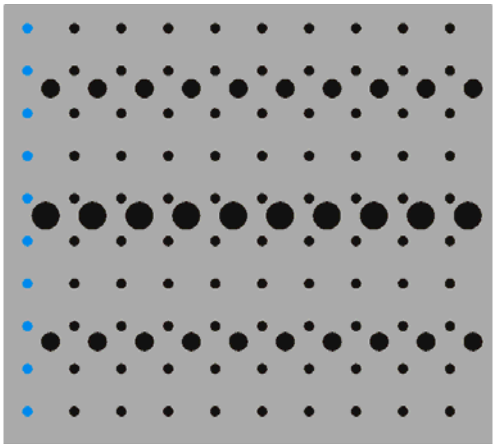

Fig. I-5: Only 10 capillaries of radius 1 have been filled with water.
The rest remain oil saturated.

We can calculate, for this saturation stage, the effective permeabilities, considering that oil-filled capillaries will only conduct oil, and water-filled capillaries will only conduct water.

- ✓ The 10 capillaries of radius 1 add effective porosity of 1% (10 x 0.1%) occupied by water. Since total porosity is 27%, water saturation is:

  Sw = 1 / 27 = **3.7 %** PV

- ✓ Knowing that Sw + So = 100% PV, oil saturation is:

  So = 100 % - 3.7 % = **96.3 %** PV

- ✓ Effective permeability to water is given by the 10 water-filled capillaries, each providing 0.1 mD. Hence:

  Kw = 10 x 0.1 mD = **1 mD**

- ✓ Thus, permeability to oil will be:

  Ko = 123 mD – 1 mD = **122 mD**

  Note: *For this model it is true that ko + kw = K. This doesn't usually happen in real porous media due to the presence of residual phases.*

- ✓ Relative permeabilities are:

  Krw = 1 mD / 123 mD = **0.0081**

  Kro = 122 mD / 123 mD = **0.9919**

Fig. I-6 shows a more advanced stage on water saturation. All the small capillaries have been filled with water. Oil remains in medium and large pores.

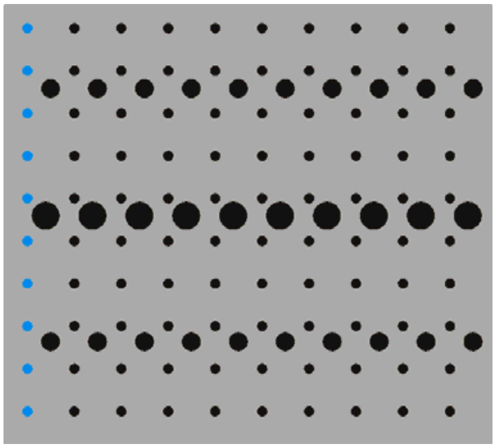

Fig. I-6:.All the small capillaries have been filled with water

Following the same procedures as for the previous case, the properties are:

- ✓ **Sw** = **37 %** PV (100 x 0.1 % / 27 %)
- ✓ **So** = **63 %** PV (100 % - 37 %)
- ✓ **Kw** = **10 mD** (100 * 0.1 mD)
- ✓ **Ko** = **113 mD** (123 mD - 10 mD)
- ✓ **Krw** = **0.081** (10 mD / 123 mD)
- ✓ **Kro** = **0.919** (113 mD / 123 mD)

> ***Note***: *At this stage in the displacement process, water invaded one third of the pore volume, but oil still held more than 90% of the flow capacity.*

Allowing the displacement process to continue, the situation reflected in Fig. I-7 is reached.

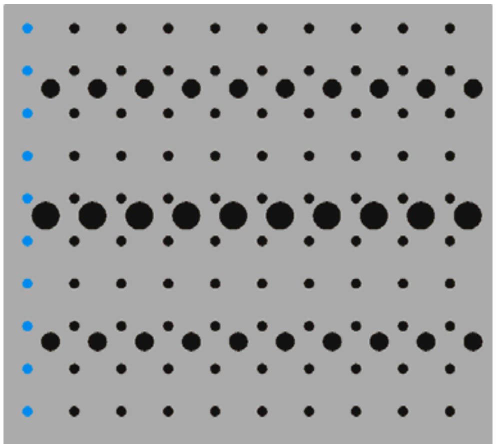

Fig. I-7: Small and medium sized capillaries are filled with water.

Again, a simple calculation leads to:

- ✓ **Sw** = **66.7 %** PV
- ✓ **So** = **33.3 %** PV
- ✓ **Kw** = **42 mD**
- ✓ **Ko** = **81 mD**
- ✓ **Krw** = **0.341**
- ✓ **Kro** = **0.659**

Plotting for different stages, Kro and Krw in the ordinate axis and Sw in the abscise, we obtain Fig I-8, which is easy to explain according to the assumed properties for the system under study:

- ✓ As the small capillaries are being filled Sw increases without any appreciable increase in the capacity to conduct water.
- ✓ Only when medium and large sized capillaries begin to be filled, an appreciable decrease in the capacity to conduct oil becomes noticeably.

> ***Note***: *Additional points where added to the curve using the same procedure detailed above.*

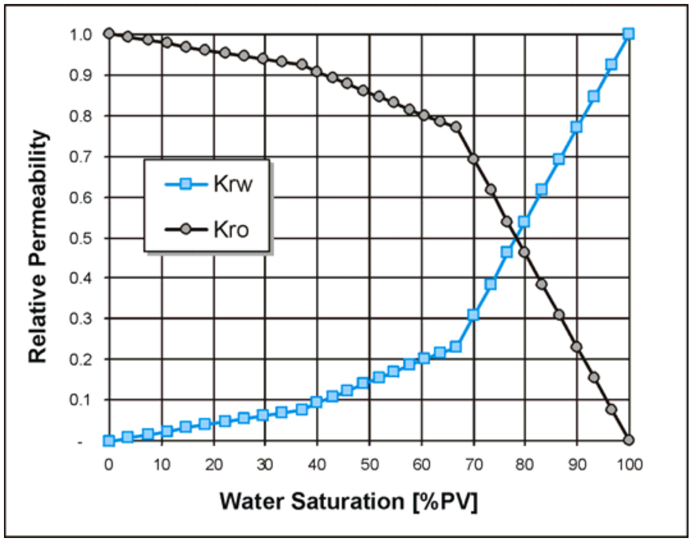

Fig. I-8: Relative Permeability Curve for a low rate displacement in a water wet system

Case II: Oil wet displacement.

Fig. I-9 shows the fill up sequence during this displacement. This case is opposite to Case I because now the first capillaries to be saturated with water are the larger ones.

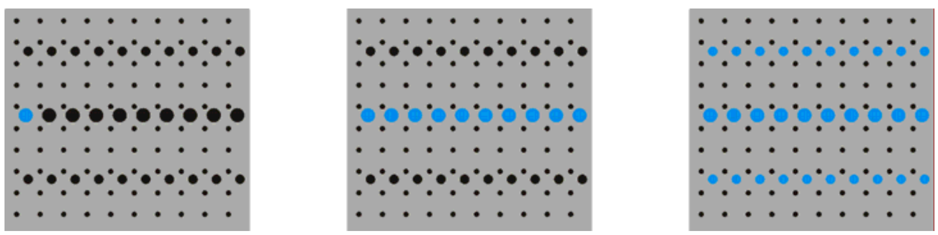

Fig. I-9: In an oil wet system, water invades large capillaries first. Further displacement leads to saturation of medium sized and small capillaries, in sequential order.

Fig. I-10 shows the relative permeability curves corresponding to case II.

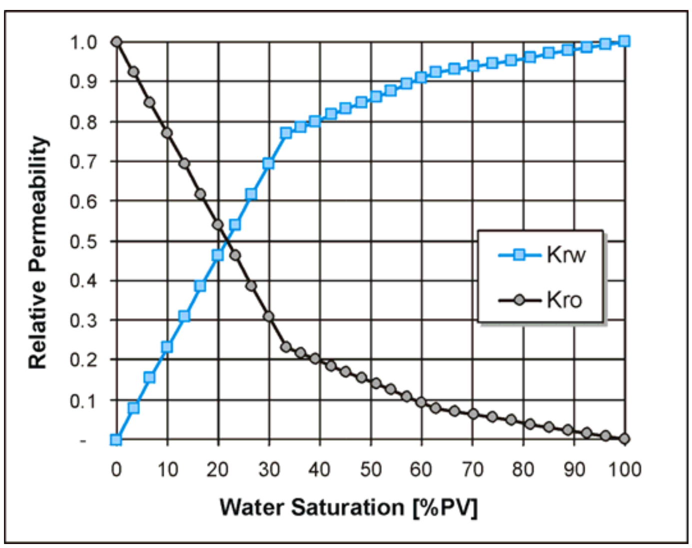

Fig. I-10: Relative permeability curves for an oil-wet system.

Case III: Displacement under gravity segregated flow conditions.

Gravitational phenomena are rarely appreciated in lab-sized sample when relative permeability curves are measured. For this reason, it is more convenient to think of our model as a small reservoir in which every level could represent a pay zone.

Water fill up is produced from bottom to top.

Fig. I-11 shows the different stages of water invasion.

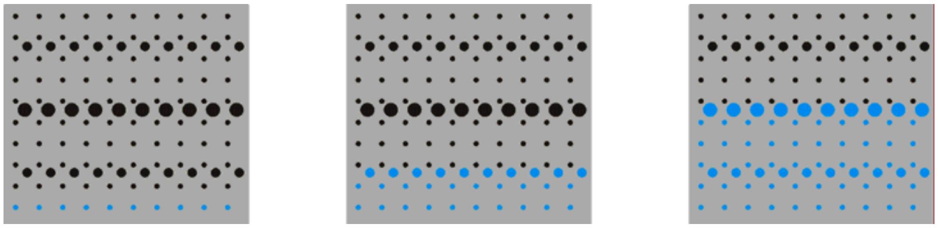

Fig. I-11: In gravity dominated systems, invasion proceeds from bottom to top regardless of the size of the capillaries in each level.

Fig. I-12 shows the relative permeabilities associated to this type of displacement.

Sudden changes in the slope of the curves are associated with the properties of the layer that is being invaded with water.

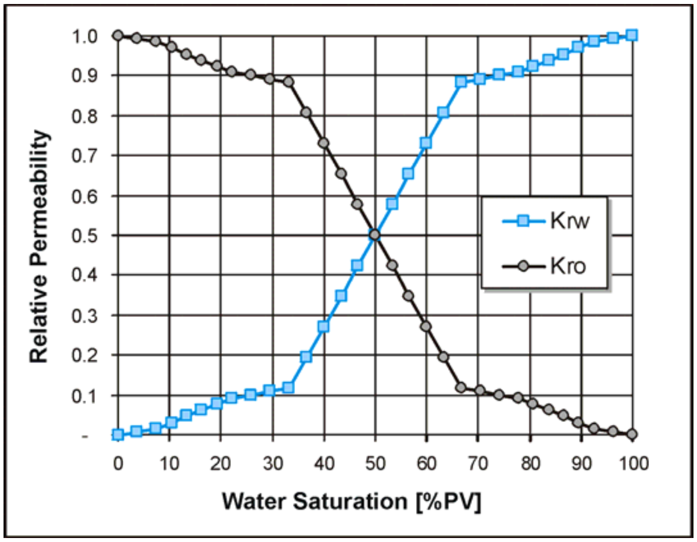

Fig. I-12: Water invasion under segregated flow conditions.

Case IV: Displacement under gravity segregated flow conditions with random distribution

Now capillaries are relocated in a random fashion throughout the section of the capillary model. In this way, as gravitational fill up proceeds, permeability to water increases steadily (Fig. I-13). This is because different sized pores are invaded in the same proportion during each water saturation increment.

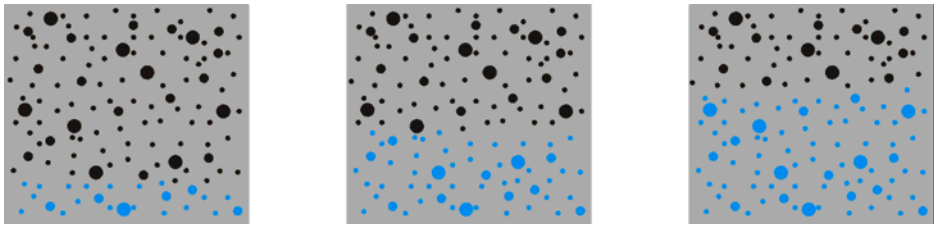

Fig. I-13: Randomly heterogeneous porous media, dominated by gravity forces. In each stage small, medium and large pores are invaded in similar proportions and therefore relative permeabilities varies steadily with Sw.

In other words, once water has taken over 25% of PV, relative permeability to water reaches 25% of its maximum value and oil relative permeability decrease to 75% of its own maximum value. (Fig. I-14).

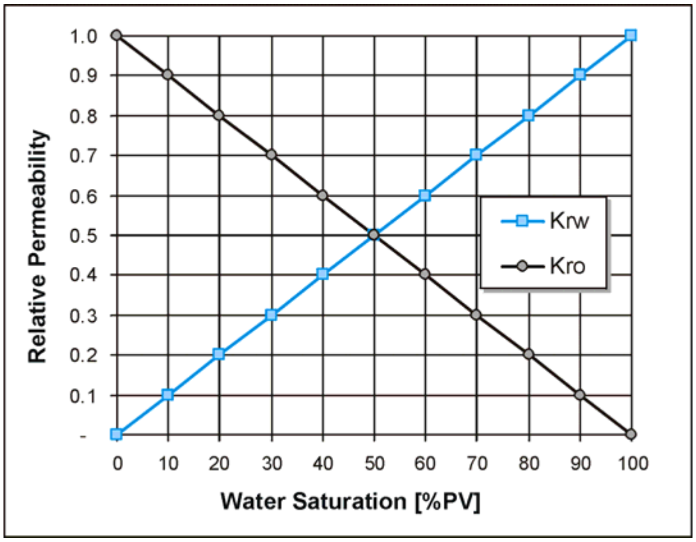

Fig. I-14: Relative permeability curves for a displacement under domain of gravity forces with randomly located capillaries

A more General Case

Each of the previously analyzed cases corresponds to a different displacement mechanism and, according to our analysis, leads to different relative permeability curves.

This situation can be generalized stating that although permeability is a property of the porous media exclusively, relative permeabilities seem to depend also on flow conditions.

Besides the pore structure, displacement mechanism plays a significant role in the saturation-flow-rate relationship.

> ***Note**: By "displacement mechanism" we refer to the equilibrium ratio between the forces that motivate the replacement of one phase by the other. These forces can be capillary (related to the interfaces between the fluid and the pore walls), gravitational (due to density difference between fluids) and viscous (originated by the application of external pressure gradients). The relative strength of these forces leads to different displacement patterns.*

REMARKS

It is convenient to revisit the assumptions we made while working with this model:

- ✓ To simplify the model, it was assumed that there were no residual phases (Sor = Swirr = 0). Yet results are qualitatively valid for systems where these conditions are not met.
- ✓ Because oil is totally displaced, the system retains the total capacity to conduct fluids (123 mD). at all times. When oil is displaced from a capillary by water, oil flow capacity becomes water flow capacity such that the total flow capacity remains constant. In naturally occurring porous media the sum of relative permeabilities is always less than 1. This is because the presence of two phases within one channel causes mutual interference.
- ✓ A water-wet system subjected to high flow rates can behave as an oil-wet system because viscous forces favor the invasion of highly-conductive large capillaries where pressure head loss is lower. Therefore Fig. I-10 can also be considered to represent a water-wet system being displaced at a high rate.
- ✓ As in usual modeling of relative permeabilities, the saturations are considered on a slice section of infinitesimal width, they are not average values over the volume of the model. As we shall see later on, this method presents severe limitations when applied to the description of the significant variables, for the petroleum engineer, in real systems: Oil production and Average saturation. This point could be assumed to be the core of the developments of the following chapters.

- ✓ Displacement under gravity segregation forces is only possible when there is vertical communication between layers. That would render our model invalid for this case. Again, the results derived can be qualitatively used to understand real systems

MAIN CONSEQUENCES

In spite of the oversimplifications made during the previous developments, the simple capillary model leads to some useful conclusions, valid for real and much more complex systems:

1. Although porosity and permeability are properties of only the porous media, relative permeabilities are not unique for a porous media but also depend, at least, on the displacement mechanism, wettability and flow rate
2. Hence (from the previous point), it is not proper to report relative permeabilities without specifying the other variables (mainly wettability & displacement mechanism)
3. Saturations and permeability values in end point conditions appear to be independent from displacement mechanisms. As discussed later, this is only a first approximation to the real word behavior.

SUMMARY OF CHAPTER I

Permeability is the capacity of a porous media to conduct fluids. It is a property of the porous media, independent of the kind of fluid circulating through it.

If many fluids are present in the porous media, the capacity of each fluid to flow depends on the type and number of capillaries occupied by each phase. This behavior is routinely described using relative permeability curves.

In the present chapter a simple model of porous medium was proposed to study the influence of displacement mechanisms on relative permeability curves.

It was established that relative permeability curves are not only a property of the porous medium, but depend heavily on the presence of capillary and gravitational forces and the magnitude of viscous forces. The same effect is observed in naturally occurring porous media.

Saturations were considered on a slice of infinitesimal width; they are not volume saturations. This value of saturation is usually referred as “point” saturation.